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FACULTY OF ENGINEERING SCIENCES

Design and Analysis of a Universal Scissor Component for Mobile Architectural Applications

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Ontwerp en Analyse van een Universele Schaarcomponent voor Mobiele Architecturale Toepassingen

Acknowledgements

In the first Master year, the course “Active Form Constructions”, under supervision of Prof. Marijke Mollaert, provided me with the opportunity to learn how to work on a project concerning deployable structures. A part of a deployable structure, including the enclosing membrane, was designed, analysed and built in real scale. Realising how these structures act differently from traditional static systems, I was very motivated to conduct more research about this subject as a master’s thesis.

The subject is wide and provides material to conduct more fundamental research on different fields. I experienced a first taste of the domain and this works very motivating. My interest in the exciting field of deployable structures made me consider a future path of further and thorough research.

I would like to sincerely thank my supervisor, Prof. Niels De Temmerman, for his continuous advice and support. He allowed me to pursue my interest in deployable structures. I have great appreciation for his encouragement and his persistence. His leadership was been invaluable.

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Lara Alegria Mira

Abstract

Deployable structures have the ability to transform themselves from a small, closed or stowed configuration to a much larger, open or deployed configuration. This dissertation focuses on the deployable structures based on pantographs or scissors.
Deployable scissor structures consist of beam elements connected by hinges, allowing them to be folded into a compact bundle for storage or transport. Subsequently, they are deployed, demonstrating a huge volume expansion. This process can be reversed, allowing re-use. In architecture, the main applications are temporary lightweight structures such as emergency shelters or exhibition and recreational structures.

Despite the advantages scissor structures have to offer, few have successfully been realised. The design process is complex and a thorough understanding is needed of the scissor geometry and its direct influence on the deployment behaviour and the structural performance.

The purpose of the research aims at designing and analyzing a new multi-configurational Universal Scissor Component. While current designs of scissor systems give an 'ad hoc' solution, this research can provide a methodology for designing a scissor component resulting in generic structures. The component will be created to develop two typologies which are of great use in architectural applications: domes and barrel vaults. A preliminary feasibility study is conducted to investigate the scissor component for these multiple structures according to geometrical, kinematical and structural implications.

Summary

In this dissertation the principles behind the design of a Universal Scissor Component (USC) are explained, resulting in a generic solution for deployable scissor structures. Based on different configurations of structures - barrel vaults and domes - decisions are made about the geometrical dimensions and the shape of the component. Considering feasible hinge positions, the design process has resulted in a USC capable of configuring nineteen different architectural structures with specific deployment behaviour.
Further, a preliminary structural study is executed to investigate the feasibility of the designed concept. A simplified method, sufficient for the projected results within the scope of this research, is implemented. The largest proposed barrel vault, with 16 units of the USC, is analysed structurally. This structure is assumed to be representative: the focus is put on critical points and a methodology is offered enabling to cover the remaining cases. Relative small sections are found, compared to the covered area, resulting in a feasible construction.
Finally, realising the importance of constructional aspects, which form the basis of the whole design of a feasible deployable structure, solutions for the joints, the deployment system and the membrane are discussed.

Samenvatting

In dit proefschrift worden de ontwerpbeginselen van een Universele Schaarcomponent (USC) toegelicht, wat resulteert in een generieke oplossing voor ontplooibare schaarstructuren. Gebaseerd op verschillende configuraties van structuren - tongewelven en koepels - worden beslissingen genomen over de geometrische afmetingen en de vorm van de component. Door haalbare scharnierposities te beschouwen, heeft het ontwerpproces geresulteerd in een USC, welke in staat is negentien verschillende architectonische structuren met een specifiek ontplooiingsgedrag te configureren.
Verder is een preliminaire structurele studie uitgevoerd om de haalbaarheid van het ontworpen concept te onderzoeken. Een vereenvoudigde methode, voldoende voor de beoogde resultaten in het kader van dit onderzoek, wordt uitgevoerd. Het grootste voorgestelde

tongewelf, met 16 USC eenheden, is structureel geanalyseerd. Deze structuur wordt verondersteld representatief te zijn: de nadruk wordt gelegd op de kritische punten en een methodologie wordt aangeboden om de resterende gevallen te dekken. Relatief kleine secties worden gevonden, in vergelijking met de overdekte ruimte, wat resulteert in een haalbare constructie.
Tot slot wordt het belang van de bouwkundige aspecten benadrukt, welke de basis vormen van het gehele ontwerp van een haalbare mobiele structuur. Oplossingen voor de scharnierverbindingen, het ontplooiingssysteem en het membraan worden besproken.

Résumé

Dans cette thèse les principes derrière la conception d'un Composant de Ciseaux Universel (USC) sont expliqués, ce qui résulte en une solution générique pour des structures de ciseaux déployables. Sur la base de différentes configurations de structures - voûtes en berceau et dômes - des décisions sont prises concernant les dimensions géométriques et la forme du composant. Considérant des positions de charnières possibles, le processus de conception a entraîné un USC capable de configurer dix-neuf différentes typologies de structures architecturales, chacune avec un comportement de déploiement spécifique.
En outre, une étude préliminaire de structure est exécutée pour étudier la faisabilité du concept. Une méthode simplifiée, suffisante pour les résultats projetés dans le cadre de cette recherche, est mise en œuvre. La plus grande voûte en berceau proposée, avec 16 unités de l'USC, est analysée structurellement. Cette structure est supposée être représentative: l'accent est mis sur les points critiques et une méthodologie est proposée permettant de couvrir les cas restants. Par rapport à la zone couverte, de relativement petites sections sont trouvées, résultant dans une construction possible.
Enfin, conscient de l'importance des aspects constructifs qui forment la base de la conception intégrale d’une structure déployable, des solutions pour les articulations, le système de déploiement et la membrane sont discutées.

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List of Symbols

a	Length of the unit line [m]
c_{ALT}	Altitude factor
c_{DIR}	Directional factor
c_{TEM}	Temporary factor
C_e	Exposure coefficient
$C_e(z_e)$	Exposure coefficient at external reference height
$C_e(z_i)$	Exposure coefficient at internal reference height
C_{pe}	External pressure coefficient
C_{pi}	Internal pressure coefficient
C_t	Temperature coefficient
d	Diameter of section [m]
e	Eccentricity [m]
E	Young modulus [Pa]
f	Optimisation factor
F	Total load [N]
F_{Ed}	The design loading on the structure [N]
F_{cr}	The elastic critical buckling load for global instability [N]
G	Permanent loads [N]
H_1, H_2	Height [m]
l_i, l'_i, k_i, k'_i	Semi-lengths of scissor beam [m]
L	Length [m]
$M_{y(,Ed)}$	Internal moment around the local y-axis [Nm]
$M_{z(,Ed)}$	Internal moment around the local z-axis [Nm]
$N_{(Ed)}$	Internal axial force [N]

q_{ref}	Reference wind pressure [N/m^2]
Q	Mobile loads [N]
R_x	Reaction force in the global x-direction [N]
R_y	Reaction force in the global y-direction [N]
R_z	Reaction force in the global z-direction [N]
s	Snow load [N/m^2]
s_k	Characteristic snow load [N/m^2]
S	Span [m]
S_{design}	Design span (at fully deployed configuration) [m]
S_{max}	Maximal span [m]
t	Thickness of section [m]
T	Structural thickness of structure [m]
U	Number of units
U_x	Displacement in the global x-direction [m]
U_y	Displacement in the global y-direction [m]
U_z	Displacement in the global z-direction [m]
v_{ref}	Reference wind pressure [N/m^2]
$v_{ref,0}$	Basic wind pressure [N/m^2]
V_z	Internal shear force in direction of the local z-axis [N]
w	Total wind pressure [N/m^2]
w_a	Weight/area [kg/m^2]
w_e	External wind pressure [N/m^2]
w_i	Internal wind pressure [N/m^2]
W_{tot}	Total weight [kg]
X	Safety factor for additional mobile loads

α_{cr}	Critical loading coefficient
β	Kink angle [°]
γ	Unit angle [°]
γ_G	Safety factor for permanent loads
γ_Q	Safety factor for mobile loads
μ_i	Form factor
θ	Deployment angle [°]
θ_{design}	Design deployment angle (fully deployed configuration) [°]
ρ	Air density [kg/m^3]
σ_{Ed}	Stress under design load [Pa]
σ_{max}	Maximal stress [Pa]

1 Introduction

To make the world work

In the shortest possible time

Through spontaneous cooperation

Without ecological offence

Or disadvantage of anyone.

—*R. Buckminster Fuller*

1.1 Deployable structures

Nowadays most constructions are static and are designed to fulfil a unique and predestined purpose during their lifetime. But in an era where nearly everything proceeds dynamically, it is interesting to explore non-static structures.

A large group of structures have the ability to transform themselves from a small, closed or stowed configuration to a much larger, open or deployed configuration. The obtained structures are generally referred to as deployable structures [Jensen, 2004].

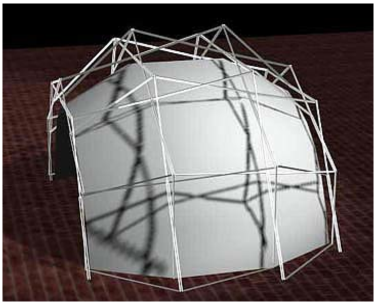
Deployables are characterized by their dual functionality as load-bearing structures or mechanisms. As load-bearing structures they transfer live and dead loads. As mechanisms they provide for the reversible alteration of their form [Rückert, 2000]. They provide the built environment with structures which can adapt to changing circumstances and requirements.

Although the research subject of deployable structures is relatively young, the principle of transformable objects and spaces has been applied throughout history (the Mongolian yurt, the pantographic weightlifting crane of Leonardo da Vinci…).
Nowadays, the main application areas are the *aerospace industry*, requiring highly compactable, lightweight payload (solar arrays), and *architecture,* requiring either fixed-location retractable roofs for sports arenas (Wimbledon) or mobile, lightweight temporary shelters (emergency tents and recreational structures).
Generally, mobile deployable structures consist of a weather protecting membrane supported by some form of erectable structure, which is capable of easily being moved in the course of normal use and can be assembled at high speed, on unprepared sites. For this purpose, scissor structures are most effective: besides being transportable, they have the great advantage of speed and ease of erection and dismantling, while offering a huge volume expansion [De Temmerman, 2007].

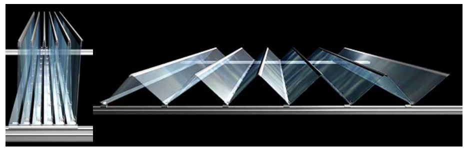
Deployable structures can be classified in four main groups according to their structural system (Figure 1.1):

a. Pantographic structures consisting of hinged beams
b. Foldable plate structures with hinged plates
c. Membrane structures
d. Tensegrity structures

This dissertation focuses on the deployable structures based on pantographs as illustrated in Figure 1.1a.

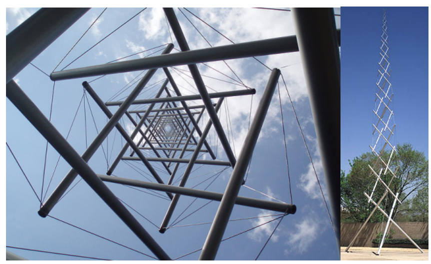


a.



b.

c.



d.

Figure 1.1: Classification of structural systems for deployable structures (a.&b. [Grupo ESTRAN c.a., 2005], c. [Structurflex Ltd., 2008], d. [Snelson])

1.2 Terminology

For clarity this section explains the terminology used in the research field of deployable structures and this dissertation.
It is scientifically correct to speak about scissor units, consisting of two beams with a revolute joint allowing a relative rotation, which are composed into 2D-linkages. Afterwards these linkages are put into a specific 3D grid generating the ultimate deployable structure.
Two different phases can be distinguished in the behaviour of deployable structures: (1) the deployment phase and (2) the service phase. In the deployment process, or transformation, the system behaves as a mechanism with generally a single kinematic degree of freedom (D.O.F.). On the other hand, the service phase, or state of application, is characterised by constraining the mechanism, which allows the system to demonstrate load-bearing behaviour. In this service phase the system is usually defined as a construction, but in literature is generally referred to as 'deployable structures'. Hereafter, the latter term will be mainly used to obtain unambiguousness.
Deployable structures might also be known as erectable, expandable, extendible, developable or unfurlable structures [Jensen, 2004].

1.3 Aims and scope of research

Despite the advantages scissor structures (or pantograph structures) can offer, few have successfully been realized. The design process is complex: a scissor structure requires a thorough understanding of the specific 2D and 3D configurations which will give rise to a fully deployable geometry. Moreover, structural implications must be considered. Flexure in the beams remains a major feature that detracts from structural efficiency [Hanaor & Levy, 2001].

The key element is that there is a direct and mutual relationship between the geometry, the kinematics and structural response of the scissor system.

The purpose of the research lies in designing and analyzing a new multi-configurational Universal Scissor Component (USC). While current designs of scissor systems give an 'ad hoc' solution, this research can provide a methodology for designing a scissor component resulting in generic structures. The designed scissor component can be used in different geometrical structure configurations and can thus be re-used meeting changing requirements.

The concept meets two important sustainable aspects. Firstly, the design is carried out in the spirit of "*Uniformity generating diversity*" as quoted by Alain Lobel. The component's equality implies mass-production which in its turn implies a cost benefit. Moreover, the diversity in forms creates multiple uses [Lobel Frames, 2005]. All of this covers the second sustainable field of "*4D*". This extension of 3D with a fourth dimension of time involves the recycle of structures by re-using materials and components for other purposes, other needs.
The USC will be created to develop two typologies which are of great use in architectural applications: domes and barrel vaults. The feasibility of the scissor component for these multiple structures is investigated according to geometrical, kinematical and structural implications.

A simplified method is applied due to a missing compatible software tool in which all components can be implemented and analyzed within the scope of a master's thesis research. For this preliminary feasibility study a combination was chosen of: (1) modelling scissor systems in the software Rhinoceros® [Rhinoceros®, 2008] and Grasshopper® [Grasshopper®, 2009] and (2) designing and analyzing scissor structures in SCIA ESA-PT [SCIA, 2007]. The method is implemented for the deployed state in which the deployable mechanisms act as constructions demonstrating a structural behaviour and a maximal loading.

1.4 Outline of master's thesis

In Chapter 2, previous work and a literature review is presented. The first part is concerned with an overview and comparison of the different scissor units. The second part focuses on what deployability implies and finally, important examples of deployable structures are mentioned.

Chapter 3 outlines the principles of the design of a specific new scissor component: the Universal Scissor Component (USC). A range of architectural configurations is chosen which determine the geometrical dimensions of the USC. Further, the deployment of the structures is discussed.

Chapter 4 deals with the methodology of the structural analysis. A representative structure is imported in SCIA ESA-PT with correct releases and climate loads to perform a steel control and an instability check and to dimension and optimise the USC.

In Chapter 5 technologies for the realisation of the deployable constructions are presented. In the light of the importance of these constructional aspects which form the basis of the whole design of a feasible deployable structure, solutions for the joints, the deployment system and the membrane are discussed.

Finally, Chapter 6 concludes this master's thesis by evaluating the proposed concepts. Also, a number of suggestions for further work are provided.

2 Review of Literature

2.1 Introduction

In this chapter a few of the main contributors to the field of deployable structures are discussed. A review of existing deployable scissor structures (or pantograph structures) is presented.

The first part covers the definitions and characteristics of traditional scissor units: translational, polar and angulated units.
Further on, the deployability condition the units have to comply with is given and implications about the kinematics of the deployment of transformable structures are discussed. In the last part, different types of examples are presented throughout developments in the domain of deployable scissor structures.

2.2 Scissor Units

Scissor units, also called scissor-like elements (SLE's) consist of two beams connected through a revolute joint, the intermediate hinge, allowing a relative rotation, but at the same time introducing bending moments in the beams. By connecting such SLE's at their end nodes by hinges, a grid structure is formed, which can be transformed from a compact bundle of elements to a fully deployed configuration. Finally, by adding constraints, the mechanism goes from the deployment phase to the service phase, in which it can bear loads.
The unit lines connect the upper and lower end nodes of a scissor unit. Depending on the location of the intermediate joint and the shape of the beams, three main unit types can be distinguished: translational, polar and angulated units.

2.2.1 Translational units

A translational unit consists of two straight identical beams linked by an intermediate hinge at the midpoints of the beams. The unit lines are parallel and remain so during the deployment. The state of deployment is characterised by the deployment angle θ.

When these units are linked, a well-known plane mechanism is formed, called a *lazy-tong.* Figure 2.1 shows a translational unit and the composed *lazy-tong* linkage. Because curvature is commonly pursued, two straight beams differing in length can form a curved translational unit (Figure 2.2).

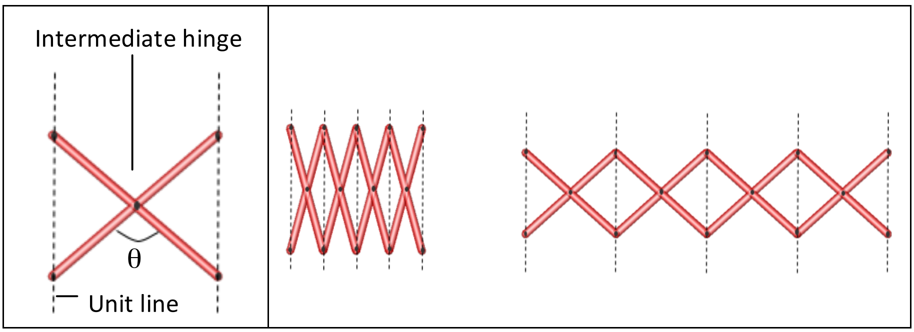


Figure 2.1: Plane translational unit and the composed *lazy-tong* scissor mechanism

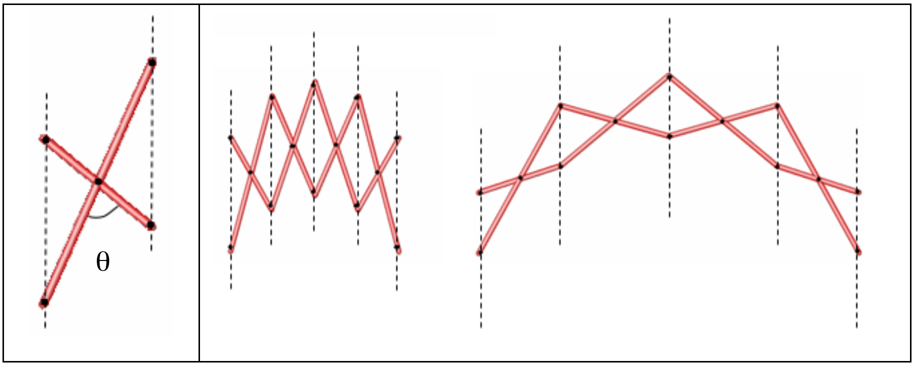


Figure 2.2: Curved translational unit and linkage

2.2.2 Polar units

Curved mechanisms can be formed by means of identical scissor units with identical straight beams by moving the intermediate hinges away from the midpoints of the beams with an eccentricity *e*. Figure 2.3 illustrates how hinge displacements have a dramatic influence on the shape. These units are referred to as polar units. The unit lines intersect at an angle γ. This angle varies strongly as the unit deploys and the intersection point moves closer to the unit as the curvature increases, as shown in Figure 2.4. During transformation, the unit lines rotate around the intersection point.

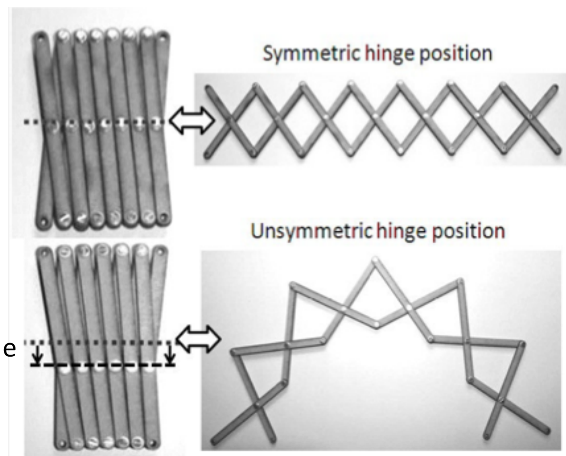


Figure 2.3: Influence of hinge displacement on the shape

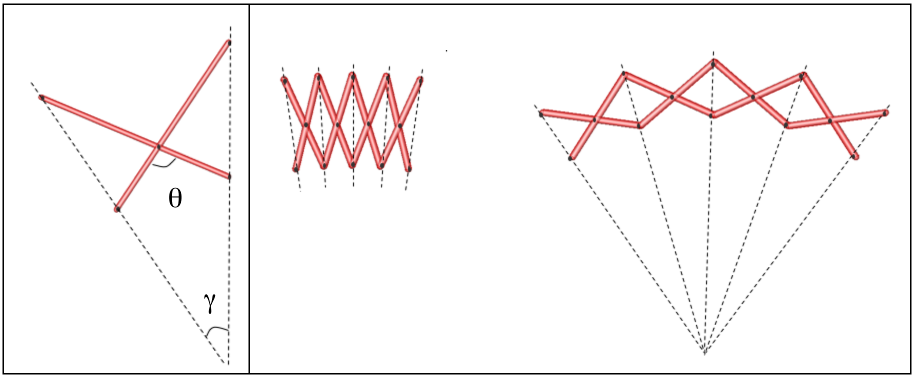


Figure 2.4: Polar unit and linkage in a closed and open deployment position

2.2.3 Angulated units

Unlike translational and polar units, which consist of straight beams, angulated elements (or hoberman's units [Hoberman, 1990]) are formed by two kinked beams with kink amplitude β as presented by Figure 2.5. As opposed to polar units, γ is constant for all values of θ if the unit geometry is such that $\alpha = \gamma/2$. This major advantage implies that only angulated units can be used for radially deploying structures with a myriad of curved shapes, capable of retracting towards their own perimeter (Figure 2.6).

Angulated elements are more multi-functional in the deployment of exotic shapes, compared to polar or translational scissors, as proven by [De Temmerman, 2007, Chapter 9] for a deployable tower (Figure 2.19d). An additional benefit of angulated scissor grids is that they can be easily triangulated (e.g. the deployable geodesic dome by Hoberman Figure 2.18) which gives them structural in-plane stiffness (non-sway characteristics) in both the deployment phase and the service phase. On the contrary, most polar and translational scissor grids, having single or double curvature, are geometrically instable during the deployment process. Only in the service phase, in the fully deployed configuration, they can be made stable by means of additional elements such as passive cables or struts (triangulation).

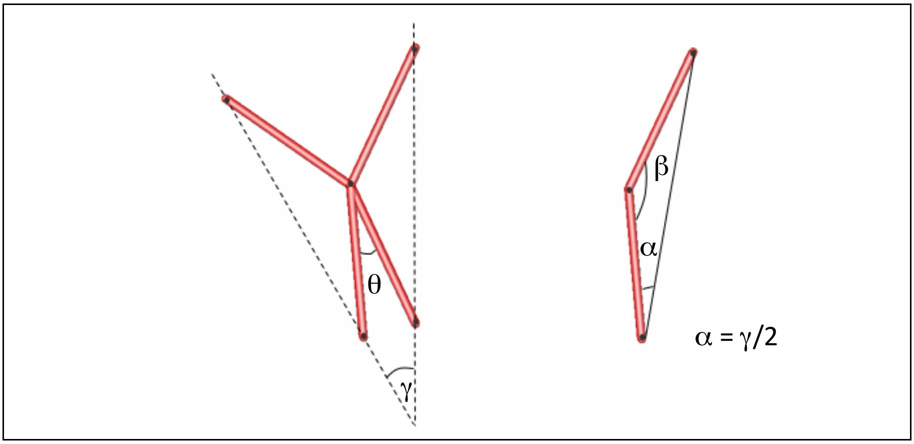


Figure 2.5: Angulated unit or hoberman's unit

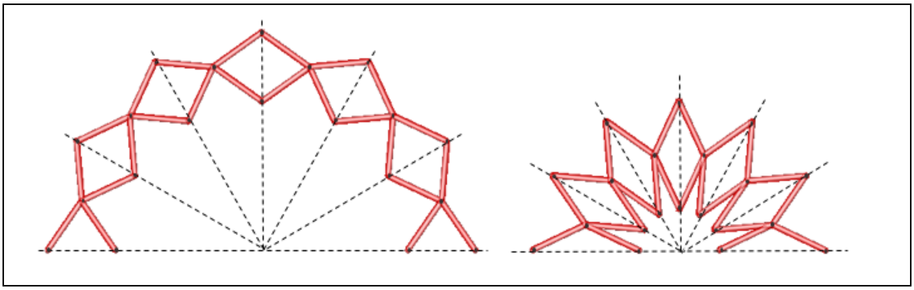


Figure 2.6: A radially deployable linkage consisting of angulated elements

You and Pellegrino [1997] extended the previous concept to multi-angulated elements. They found that two or more such retractable structures could be joined through the scissor hinges at the element ends. Two angulated elements, from layers that turn in the same sense of two such interconnected structures, with a scissor hinge in common were found to maintain a constant angle during the transformation of the structure. Hence they could be rigidly connected to each other, thus forming a single multi-angulated element with more than one kink as shown in Figure 2.7 and 2.8.

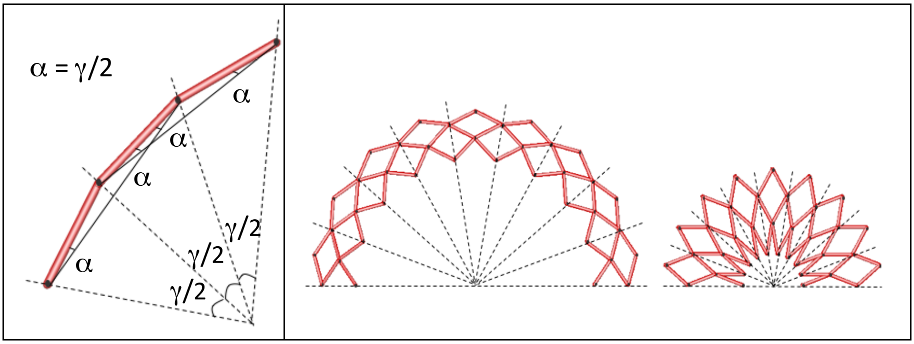


Figure 2.7: Multi-angulated unit and linkage

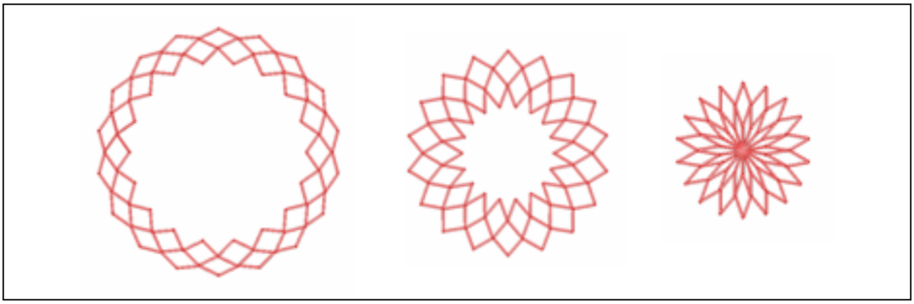


Figure 2.8: A radially deployable linkage of multi-angulated elements in three stages of deployment

2.3 Kinematics of the deployment

2.3.1 Deployability constraint

The deployment of a structure is a transformation which is ensured by the introduction of kinematic degrees of freedom. Crucial to the design of deployable scissor units is the deployability constraint. This formula is derived by Escrig [1985]:

$$l_i + l'_i = k_{i+1} + k'_{i+1} \quad (2.1)$$

The constraint implies a necessary and sufficient condition to guarantee folding and expanding: the sum of the semi-lengths of a scissor unit has to be equal to the sum of the semi-lengths of the adjoining unit in all cases (Figure 2.9). Langbecker [1999] extended the deployability condition of Escrig to include three-dimensional configurations.

It should be noted that scissor linkages which do not comply with Equation 2.1 can still be partially foldable [De Temmerman, 2007]. However, since this dissertation concerns the design of compactly foldable scissor structures, the deployability constraint is treated as a minimum requirement.

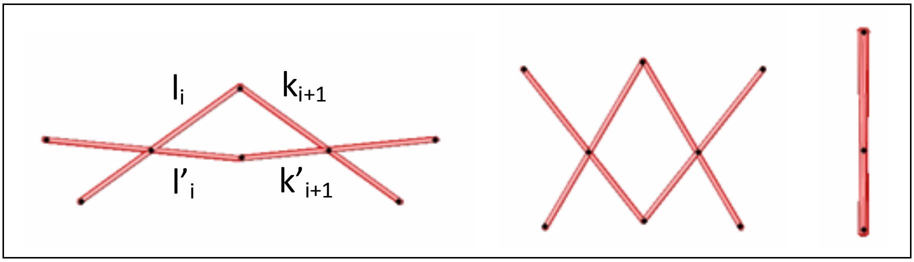


Figure 2.9: The deployability constraint in terms of semi-lengths in three consecutive deployment stages

2.3.2 Snap-through effect and bi-stable structures

Due to geometric incompatibilities in length of elements, stresses and strains can occur during the transformation. This moment is called snap-through and locks the structure in the deployed configuration.

If stresses and strains are also detected in the fully closed and deployed situation, then the bars are increasingly sensitive to buckling leading to a decrease of the total loading capacity of the system. This situation can be avoided by choosing a good geometric design that complies with the terms of non-linear algebra and trigonometric equations related to the straightness of the beams in the two extreme situations [Gantes, 1997]. Zeigler [1981] was the first one to exploit this phenomenon as a self-locking effect.

If the elements are in a stress-free state before and after deployment, but go through an intermediate stage with deployment-induced stresses, they are called bi-stable deployable structures. Figure 2.10 presents such a bi-stable structure. The diagonal units (red) are subject to elastic deformation in the intermediate deployment stage. The structural response of a system with snap-through during transformation is studied with a sophisticated final element model.

This research avoids snap-through situations and focuses on mechanisms.

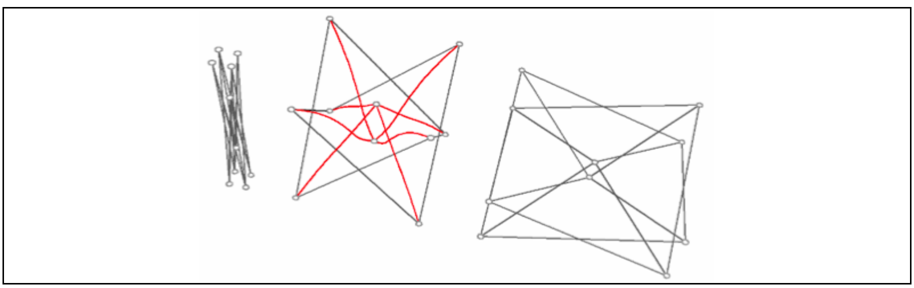


Figure 2.10: Bi-stable structure before, during and after deployment [De Temmerman, 2007]

2.4 Deployable structures

Work by Spanish architect Emilio Perez Piñero introduced the application of scissors structures. Although previous concepts for deployable structures existed before Piñero, his structures were the first complex deployable systems used to cover large areas. Beginning in 1961, he developed numerous structural designs such as his moveable theatre, presented in Figure 2.11. His structures took on both flat and domical shapes. Piñero's shelters were foldable due to their strain-free deployment, behaving as mechanisms. His work in the field of deployable structures attracted interest to the study field and produced one of the few practical realisations of modern deployable structure design [Melin, 2004].

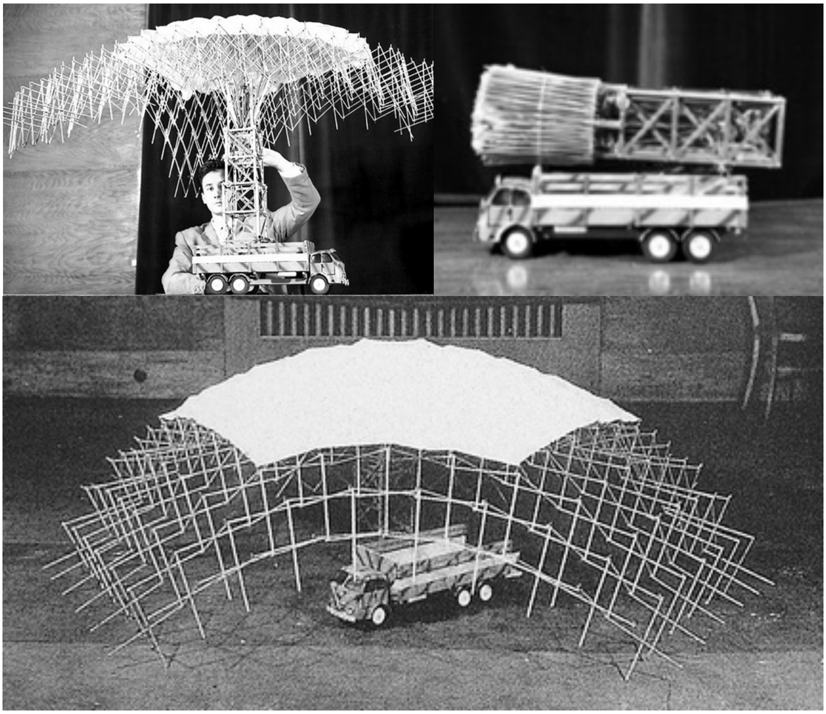


Figure 2.11: Piñero and his pantographic dome [Fundacion Emilio Perez Pinero]

In more recent years, another Spanish architect became one of the main contributors to the field. As mentioned in section 2.3.1 Felix Escrig [1985, 1993] presented the geometric condition for deployability. In many years of research and with the help of colleagues, such as J.P. Valcarel, he demonstrated how three-dimensional structures could be obtained by placing translational and polar units in multiple directions on a grid. Figures 2.12, 2.13 and 2.14 present a few of unnumbered proposed geometric models.

One of Escrig's notable applications was the development of a deployable cover for the San Pablo Sports Centre in Seville, Spain. The aim was to develop a deployable structure to cover a swimming pool to allow use during the winter. The design consists of two identical rhomboid grid structures with spherical curvature (Figure 2.15).

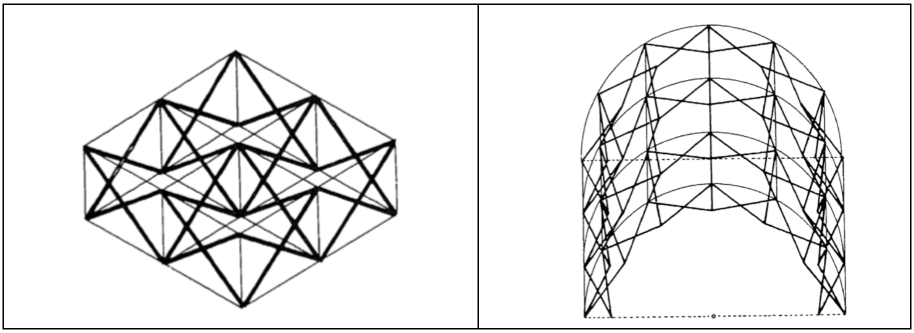


Figure 2.12: Two way grid: planar with translational units (left) and cylindrical with polar units (right) [Escrig, 1985]

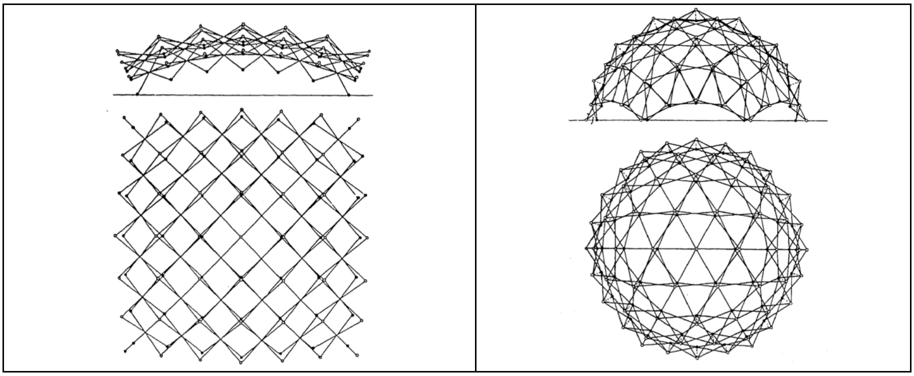


Figure 2.13: Side elevation and top view of a two way spherical grid (left) and a three way spherical grid (right) with polar units [Escrig, 1993]

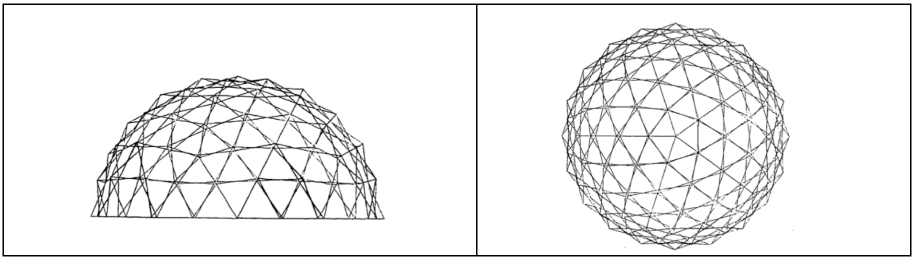


Figure 2.14: Side elevation and top view of a geodesic grid with polar units [Escrig, 1993]

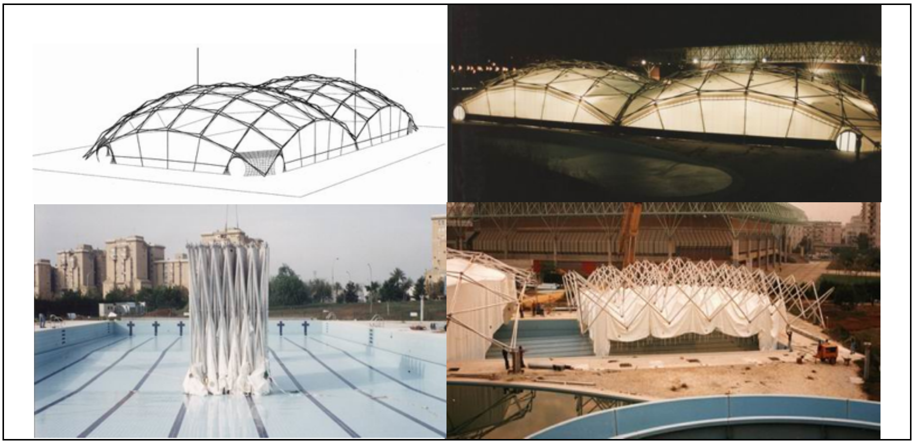


Figure 2.15: Deployable cover for swimming pool in Seville designed by Escrig & Sanchez [Performance S.L.]

Charis Gantes [1997, 2001] investigated systematically bi-stable or snap-through structures and has developed a geometric design approach for flat grids, curved grids and structures with arbitrary geometry. He conducted also research on the structural response during the deployment which is characterised by geometric non-linearities. Simulation of the deployment is therefore an important part in the analysis requiring sophisticated finite element modelling.

As mentioned, Hoberman [1990] introduced a new concept in the field of scissor structures: angulated elements. The ability has risen to create radially deploying closed loop structures, which are impossible to accomplish with translational and polar units, because these demonstrate a linear deployment. You & Pellegrino [1997] extended the concept to multi-angulated elements.

Kassabian [1997] implemented multi-angulated elements by providing them with cover elements resulting in a retractable roof as shown in Figure 2.16. The cover elements accomplish both in the open and closed position a gap-free, weatherproof surface.
Instead of covering a bar structure with plates, Jensen [2004] has proven that the angulated elements could be removed, connecting the cover elements directly to one another at exactly the same locations as in the original bar structure. This material improvement does not change the kinematic behaviour of the expandable structure.

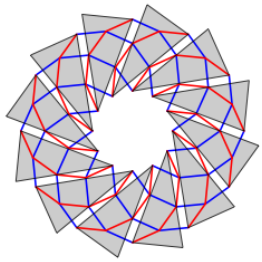


Figure 2.16: Multi-angulated structure with cover elements in an intermediate state of deployment [Kassabian, 1997]

Several researches were intrigued by the wide range of possibilities of dome shaped structures. So was Hoberman, inspired by the architectural philosophy of Buckminster Fuller, pioneer in dome geometries and he is called “the 20th-century Leonardo da Vinci” [Baldwin, 1996].

Angulated elements connected by scissor hinges not only subtend a constant angle in a plane surface, but also on a conical surface. This was discovered by Hoberman [1991], who has proposed the Iris Dome.

Figure 2.17 shows a sequence of views of an iris-type retractable structure having an oval-shaped perimeter and cover plates attached to it. Hoberman exhibited a cover-less model for the Iris Dome at the Expo 2000 in Hannover. Unlike the Iris Dome which is a concept of a deployable mechanism at fixed location, Hoberman’s Expanding Geodesic Dome is a transportable structure (Figure 2.18). It blossoms open from a 1,5-meter cluster to a 6-meter dome when pulled open from its base. When deployed it has the same shape and triangulated pattern as Buckminster Fuller's static, geodesic dome, taking this seminal historic structure into the 21st century.

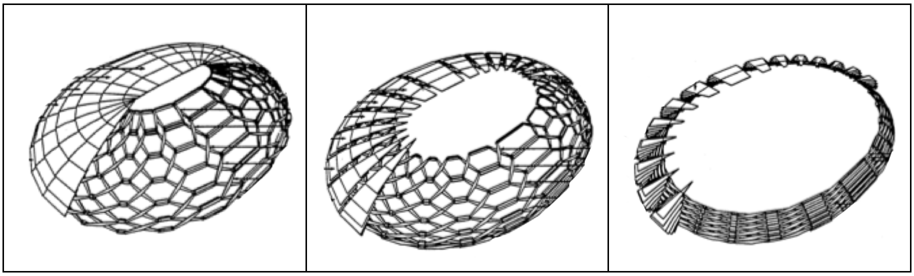


Figure 2.17: Iris-type retractable structure in different deployment stages [Hoberman, 1991]

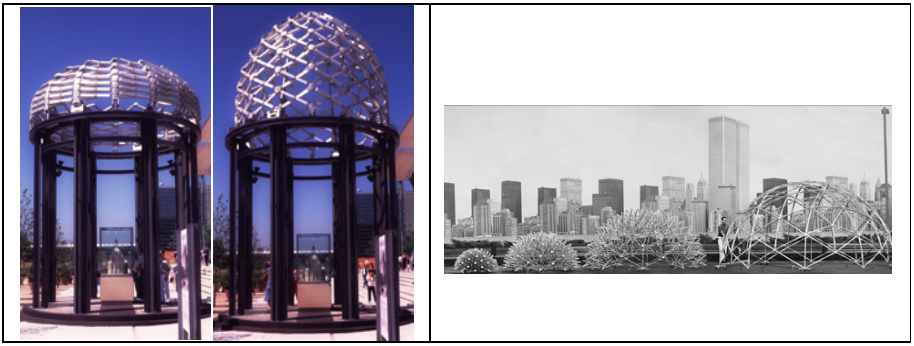


Figure 2.18: Retractable dome on Expo Hannover – Expanding Geodesic Dome [Hoberman Associates Inc.©]

Recently in 2007, Niels De Temmerman [De Temmerman, 2007] has presented novel concepts for deployable bar structures and proposed variations of existing concepts, which lead to architecturally, as well as structurally viable solutions for mobile applications (Figure 2.19).

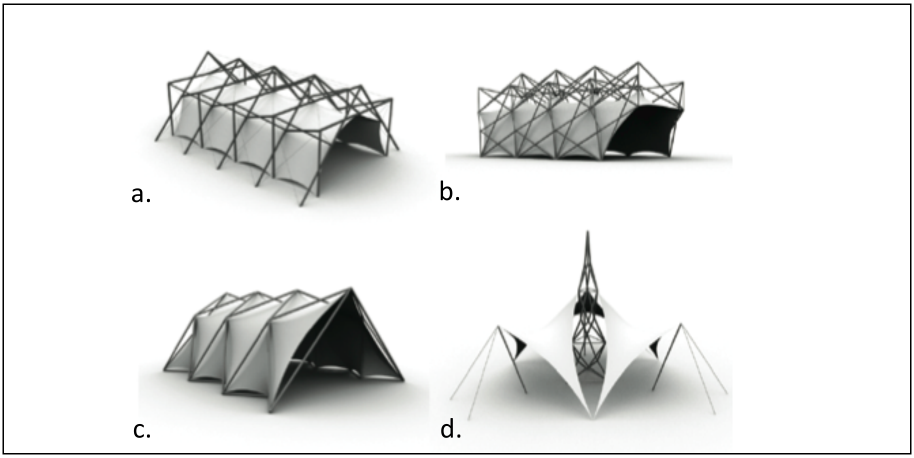


Figure 2.19: a. & b. Two deployable structures with polar and translational units – c. Deployable structure with articulated bars – d. Deployable mast with angulated units [De Temmerman, 2007]

3 Geometric Design of a Universal Scissor Component (USC)

3.1 Introduction

This chapter is concerned with the geometric design of a new type of scissor component. The current methodology for creating and designing a structure is characterised by an 'ad hoc' solution. A structure is built to meet a specific need or a single set of requirements. However, changing needs in our dynamic lives are the stimulation to create structures that can be reconfigured and adapted to evolving circumstances, within a society that embraces the concept of 'sustainability'.

This research provides a 'generic' solution for the design problem: one single unit is used to create a multitude of geometrical configurations for deployable scissor structures.
Hanaor & Levy [2001] discussed the design and evaluation criteria for existing deployable structures. In an analogue way Table 3.1 illustrates the possibilities and advantages of the new concept of the Universal Scissor Component (USC) in the design phase, behaving structurally different compared to traditional scissor units (cfr. section 3.2.1).

Table 3.1: Evaluation criteria in the design phase for a USC in deployable structures

Criterion	Structure-morphological aspects and comments
Architectural flexibility	- Design enabling multi-purpose applications - Periodic structures which can be re-used - Generic design for multiple geometrical configurations
Component uniformity	- Uniformity creating diversity in possible configurations - Standardisation of component (affects manufacturing costs)

The first part of this chapter is concerned with the geometric design of the USC. The geometrical form and dimensions are inspected and the multiple architectural structures are discussed. The software tools Rhinoceros® and Grasshopper® are used to create the architectural geometries. In a second part the kinematic implications are investigated. Here also, Rhinoceros® and Grasshopper® served as tools to explore the deployment properties.

3.2 Design of the Universal Scissor Component

3.2.1 Structural background

A first indication about an efficient geometrical shape can be found through a structural analysis of different existing scissor units, as described in paragraph 2.2.

A translational, polar, angulated unit and a triangular component are compared structurally. The units are fixed at their lower nodes with hinged supports and loaded with point forces of 0,5kN in their upper nodes as indicated in Figure 3.1.

The reason these units are examined is to form an idea about the structural behaviour of the three main elements (Figure 3.1a,b,c) and to investigate which structural implications there can be distinguished if a new triangular shape is formed (Figure 3.1d). The triangular component can be seen as a combination of a straight translational/polar beam (Figure 3.1a,b) and an angulated kinked beam (Figure 3.1c). The means of triangulation plays in favour of the design concept of a unique universal scissor unit.

The internal forces in the beams are presented in Figure 3.2, 3.3 and 3.4.

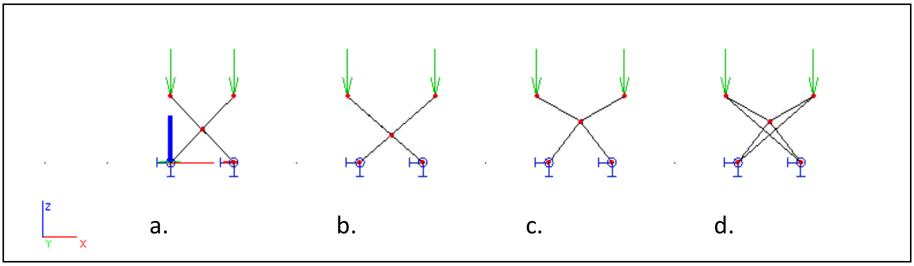


Figure 3.1: Structural models of the scissor units in SCIA ESA-PT: a. translational – b. polar – c. angulated – d. triangular

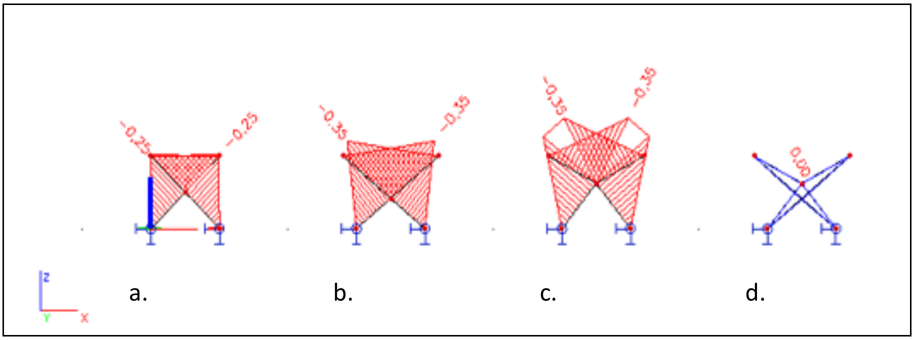


Figure 3.2: Diagrams of bending moment M_y [kNm]

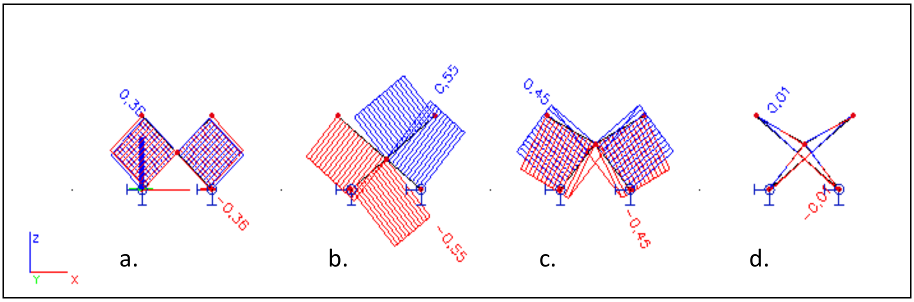


Figure 3.3: Diagrams of shear force V_z [kN]

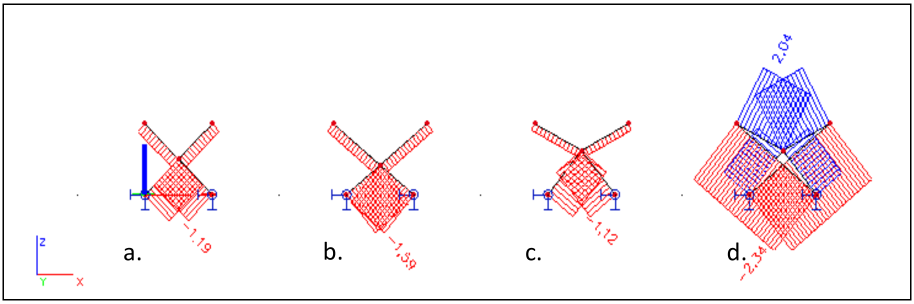


Figure 3.4: Diagrams of axial force N [kN]

This preliminary structural study, where only the reduction forces are analysed, provides interesting information about the different inspected units.
If the bending moments (Figure 3.2) are considered, the behaviour of the main scissors (a, b, c) is comparable. In case of the triangular component (d) a significant reduction of M_y is observed. The same conclusion is valid for the diagrams of the shear force V_z (Figure 3.3). The opposite is distinguished when considering the axial forces (Figure 3.4): larger values occur in the beam elements of the triangular unit.

The conclusion can be made that the bending moment M_y and shear force V_z are reduced to zero or seeming zero in case of the triangular component. This indicates a larger efficient use of material compared to the standard scissor units in which undesired bending moments are distinguished . Also, a triangle is known as a geometrically indeformable shape, providing a larger stiffness.
The remark can be made that, compared to the standard scissor units, the triangular component requires more material and thus results in a higher weight. But this disadvantage is considered to be of lesser significance than the obtained advantage of a generic design solution as will be proven in the following sections.

3.2.2 Configurations of the structures

To determine the geometrical dimensions of the USC, a closer look has to be taken which configurations of structures to consider. Deployable structures with a function of a temporary shelter have to form a three-dimensional space.
As clarified in section 1.2, dealing with terminology, the configurations which will be discussed are theoretically mechanisms. However they will be referred to as structures, because ultimately they will be used as such in an architectural environment and become stabilised in order to carry loads.

3.2.2.1 Barrel vaults

A large number of configurations are possible for three-dimensional scissor structures composed of translational and polar units, all obeying the deployability constraint. However, only a part of those will be stress-free deployable before, during and after deployment, effectively behaving like a mechanism (and thus not demonstrating any snap-through behaviour). For a classification of which three-dimensional grid structures built from translational and polar units are stress-free deployable the reader is referred to [De Temmerman, 2007, Chapter 3].

To perform an architectural function (providing weather protection) a barrel vault is a simple, but effective typology. It is chosen because an easy to govern shape is preferred to inspect a first application of the USC. Barrel vaults or cylindrical grids are monoclastic shapes. They can be obtained by curving one direction of an orthogonal two-way grid.

Using polar units is an effective way of introducing single curvature in an orthogonal grid as shown in Figure 3.5:

- direction X, or transverse direction, contains rows of identical polar units in arch formation
- direction Y, or longitudinal direction, contains parallel rows of identical plane translational units connecting the polar arches

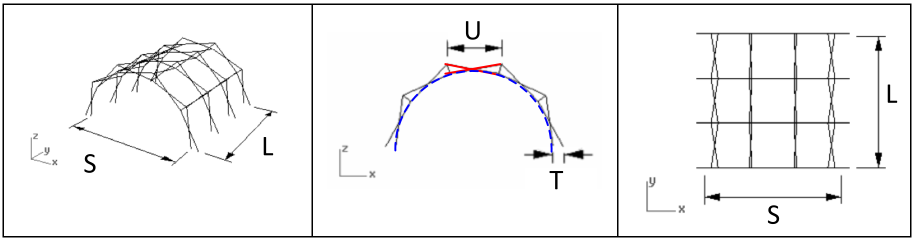


Figure 3.5: Perspective, section and plan view of a polar barrel vault

The geometric construction is done in Grasshopper®, which is a graphical algorithm editor tightly integrated with 3D modelling tools of Rhinoceros®. Using Grasshopper® the geometric design process is facilitated by developing scripts which generate the desired structures. The geometry is based on a chosen circular arch (marked in a blue dotted line in Figure 3.5) and characterised by the main architectural parameters: Span (*S*), number of polar Units (*U*) and structural Thickness (*T*). The geometric construction is based on the methodology explained in [De Temmerman, 2007]. The global length (*L*) in the Y direction is determined by the desired number of polar rows.

Obeying the deployability constraint, an adjoining polar and translational unit must have the same sum of semi-lengths. This implies that the length of a polar and translational beam is identical in the inspected barrel vault

type. This plays in favour of the concept of designing a unique scissor component for plural configurations.
Figure 3.5 is an illustration of a basic stress-free deployable single curvature structure. This kind of structures is referred to as polar barrel vaults. Because these are the only barrel vault structures examined in this dissertation, they will be further referred to as barrel vaults.

3.2.2.2 Domes

Besides the simple but effective barrel vaults, also dome geometries are considered in this research. Domes are not only architectural and structural viable structures, they can also serve as a geometric transition to more exotic and interesting shapes thanks to angulated elements.

All domes share certain advantages. Their compound-curved shape is inherently strong, giving a self-supporting clear span with no columns. Domes are resource and energy-efficient because, of all possible shapes, a sphere contains the most volume with the least surface. Thus, for a given amount of material, a dome encloses more floor area and interior volume than any other shape [Baldwin, 1996].

With the introduction of angulated elements, the ability to create radially deploying closed loop structures has risen. The multi-functionality and benefits of angulated units, as discussed in paragraph 2.2.3, have encouraged the research on deployable dome structures, based on a spherical space. Compared to translational and polar elements, angulated units can be easily triangulated and they offer the ability to compose a wide variety of grid geometries. In addition, during deployment the overall geometrical shape does not change, only a variation in span is noticed (Figure 2.18 – Expanding Geodesic Dome).

To minimize distortion of two- or three-way grids over a sphere, polyhedra can be used. Deployable grids are in this case obtained by substituting every edge of a polyhedron by scissors satisfying the compatibility conditions. Moreover the upper and lower nodes of the scissor units are aligned to the centre of the enclosing sphere of the polyhedron [Escrig & Valcarel, 1993].

The five regular polyhedra or 'Platonic' solids are presented in Figure 3.6. These are the only five polyhedra with identical faces, the same number of faces coming together at each vertex and identical edge lengths. If constructed with struts including unreinforced vertices, only the tetrahedron, octahedron and icosahedron are stable, because of the triangular faces. By raising the 'frequency' of a polyhedron, the edges are subdivided into shorter segments and smaller faces, to make the polyhedron more spherical (Figure 3.7). At all times, all the vertices touch the inner surface of the enclosing sphere.

In this research the possibility of increasing the frequency is not considered, due to the fact that this would result in different lengths of the edges which are to substitute with a scissor element. Identical edge lengths are desired, because an identical USC in the whole global structure is the starting point.

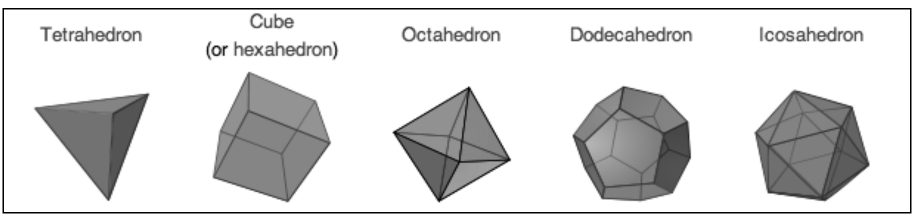


Figure 3.6: The Platonic Solids [Wikipedia®, 2010]

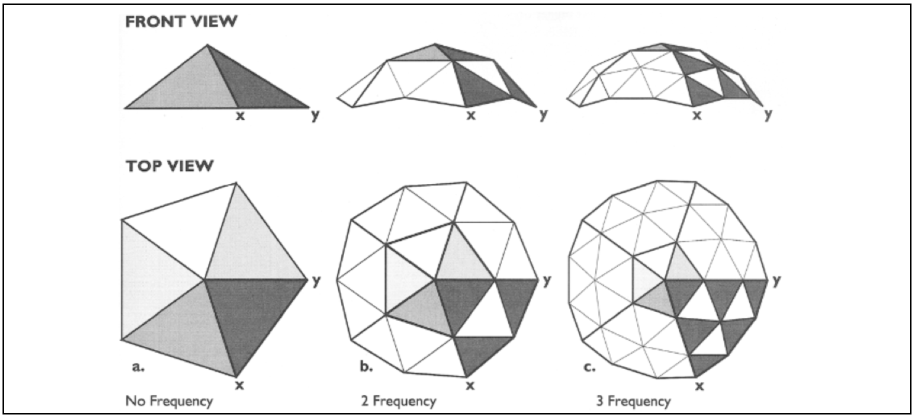


Figure 3.7: Increasing the frequency of triangulation in case of an icosahedron [Baldwin, 1996]

In this master's dissertation the following polyhedra are considered for the dome structures: icosahedron, dodecahedron, icosidodecahedron, 'buckyball' (a spherical fullerene[1] or truncated icosahedron) and an adjusted rhombic triacontahedron. Figure 3.8 illustrates the evolution of a dodecahedron to an icosahedron by truncating the vertices. In this evolution the icosidodecahedron and the buckyball appear. The rhombic triacontahedron is adjusted in such way that all the vertices touch the surface of the enclosing sphere. By doing so, all the edges become equal in length.

This selection of random polyhedra results in a multitude of different geometries for the architectural dome structures investigated.

[1] The name was an homage to Buckminster Fuller, whose geodesic domes it resembles.

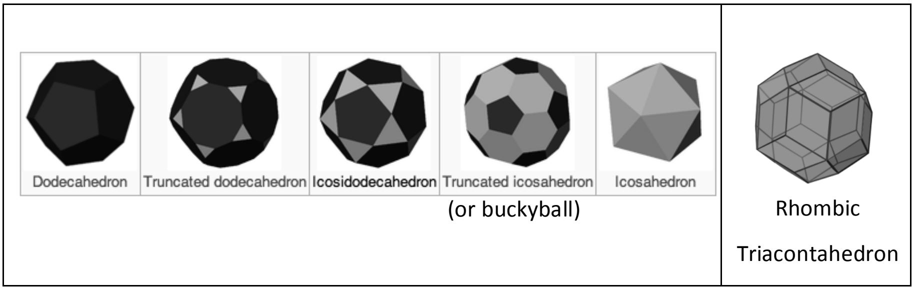


Figure 3.8: The considered polyhedra for the dome structures [Wikipedia®, 2010]

A dome structure, independent from the considered polyhedron, is made deployable by substituting every edge of the polyhedron by scissor elements. For this purpose angulated scissors are implemented because of their beneficial properties, such as the ability to develop a more stabile deployment process for shapes capable of retracting towards their proper perimeter. Figure 3.9 shows how the adjusted rhombic triacontahedron forms the basis of a deployable dome using angulated scissors.

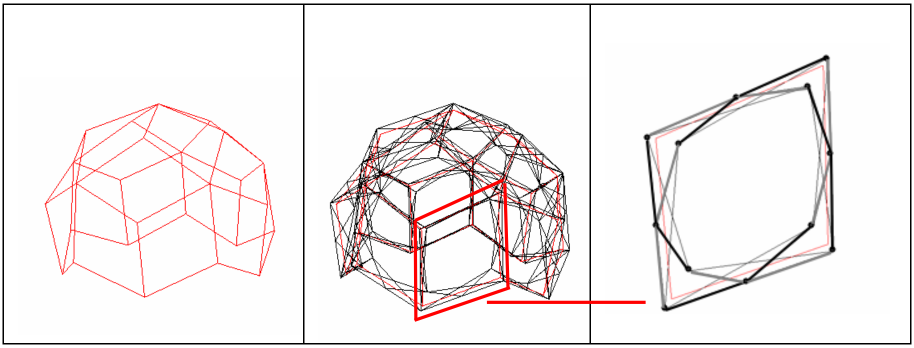


Figure 3.9: The adjusted rhombic triacontahedron - deployable dome structure with scissor elements – detail of edge replacement by angulated units

3.2.3 Geometrical dimensions

In the previous section the configurations of the structures were discussed. A USC will be designed with the ability to configure both barrel vaults as domes. To reach this possibility, the USC must be able to function as the three standard scissor units: translational, polar and angulated, depending on the desired end configuration.
Because hinge displacements have a dramatic influence on the structure shape, in this section decisions will be made concerning the different geometrical dimensions based on possible hinge positions. These are the dominating aspect for the geometry: because all the components are identical, the only difference between the configurations is the position of the pivot hinge.

3.2.3.1 Angulated part of the component

The angulated element or the kink angle is determined by the different geometrical polyhedra to form radially deployable domes. As mentioned before, the vertices of the polyhedra touch the surface of the enclosing sphere, which is inherent to each polyhedron (Figure 3.11). Every edge of a considered polyhedron is replaced by an angulated scissor.
If the fully deployed state of a scissor is considered, which implies that the two angulated beams coincide ($\theta = \beta$), all the end nodes of the angulated units touch the surface of the sphere. In this state the dimensions of the angulated part of the USC can be determined according to the edge length.

Two possibilities are considered: a polyhedron edge replacement by (1) a single or (2) a double scissor unit. The implications of these two possibilities are elucidated by means of the parameter on which will be anticipated, namely the length of the angulated element, as defined in Figure 3.10. In case of a single unit per edge, the length of the angulated scissor is equal to the edge length of the polyhedron resulting in a specific dome span. If a double scissor unit is considered per edge, the angulated length establishes the half-length of the polyhedron edge, which creates a larger dome span compared to the case of a single component. The two cases are illustrated in Figure 3.12 (a. single unit – b. double unit).

Not only these two options determine the ultimate span of the dome, they also implicate different dimensions of the angulated part of the component as shown is Figure 3.10. Two heights are distinguished: H_1 of the element in case of one scissor unit per edge and H_2 in case of replacing an edge length by two scissors. These heights are fixed by the enclosing sphere of the polyhedron and thus the edge length. The combination of length and height will determine a unique kink angle (Figure 3.13).

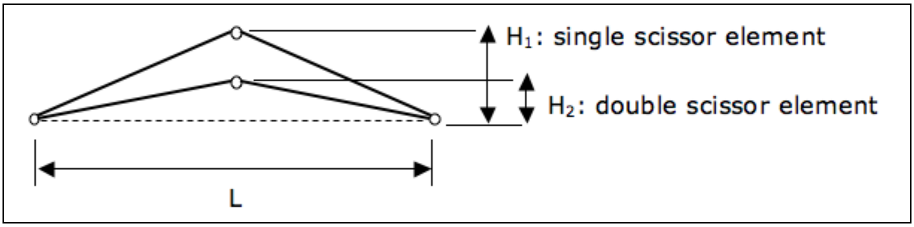


Figure 3.10: The parameters of the angulated unit in case of single or double elements

The following methodology is applied for every polyhedron to determine the geometrical dimensions and will be presented with the example of the adjusted rhombic triacontahedron.

In a first step the enclosing sphere is defined (Figure 3.11). Secondly, based on the centre point of the sphere and two intersecting vertices of an edge, the angulated unit can be constructed as illustrated in Figure 3.12 a&b, where the red lines are formed by the centre point and two intersecting vertices. The unit angle γ determines α, because $\alpha = \gamma/2$ (cfr. paragraph 2.2.3). With the definition of α, the angulated element is formed. Now, the ratio between the length and the height of a specific angulated unit is known (Figure 3.13). Finally, assigning a specific length to the angulated component, resulting in a certain height, determines the overall span of the considered dome.

In case of double scissors per edge, the same process is applied with the only difference that the angle γ is determined by the bisector as shown in Figure 3.12b.

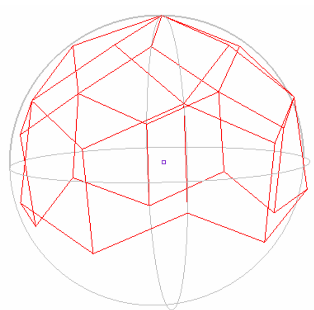


Figure 3.11: The adjusted rhombic triacontahedron polyhedron and the enclosing sphere

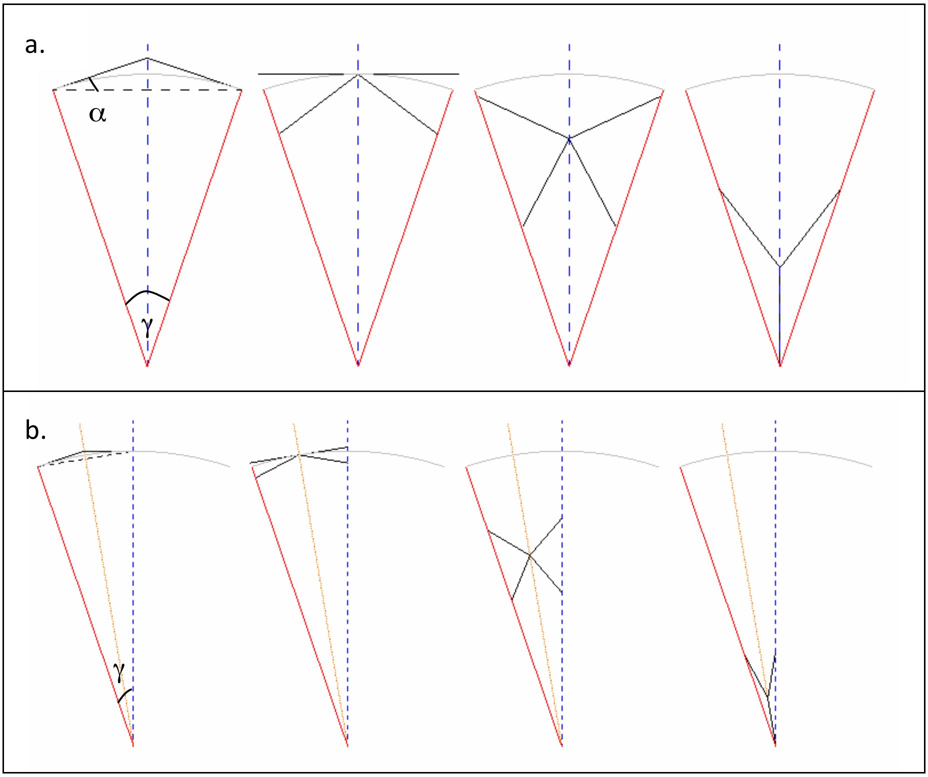


Figure 3.12: The construction of the angulated unit with the evolution of the element from the fully deployed to the undeployed state (a. single unit – b. double unit)

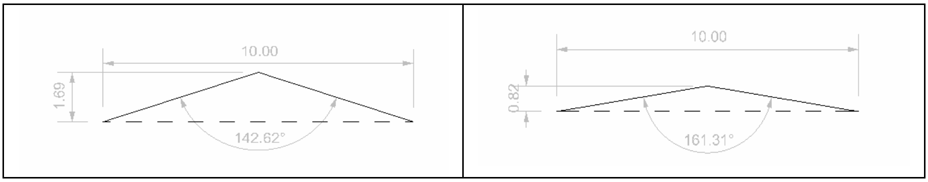


Figure 3.13: Example of the relation between the different geometrical dimensions of an angulated unit in case of a single (left) and double (right) edge replacement

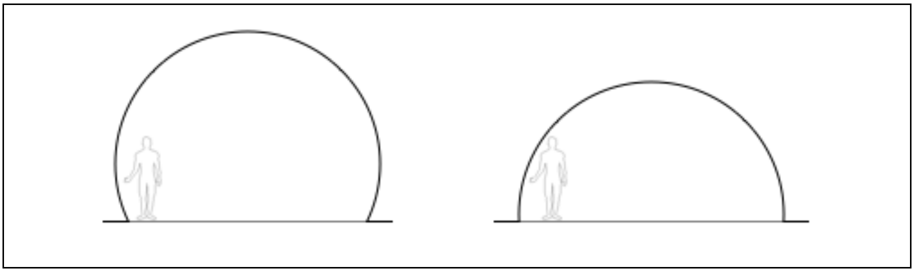


Figure 3.14: Lower domes result in less headroom at the edges

In Appendix 1 a full overview is given of the dimensions of the angulated elements and the spans of the resulting dome for each type of polyhedron[2]. Generally, the constructed space of a dome consists of more than half a sphere, a 5/8 dome[3]. The reason lies in a gain of headroom at the edges, as presented in Figure 3.14. In the appendix, additional information is given in case of possible lower constructed domes.

As presented in Appendix 1, different lengths for the angulated element are inspected. A length of 2m is chosen because of the resulting range from low to high spans for multifunctional deployable structures. Moreover, a length of 2m seems to still be manageable manually. The next step is to choose different height values. This decision is based on the feasibility of the distance between the angulated intermediate hinge positions. For that reason the icosidodecahedron is excluded. Figure 3.15 presents the ultimate geometric dimensions for the angulated part of the USC for which six different dome structures can be built geometrically.

[2] The spans will be smaller than the theoretical values indicated in Appendix 1. These theoretic spans result from the fully deployed state of the angulated elements. In real-life applications the units are not fully deployed and show a certain structural thickness.

[3] 5/8 is not an exact number, it is mainly used as a reference.

	Hinge position [cm]	Dome polyhedron	Elements per edge	Span Dome [m]
1	10,2	Buckyball	Double	16,9
2	16,4	Adjusted rhombic triacontahedron	Double	12,2
3	20,6	Buckyball	Single	8,5
4	28,4	Icosahedron	Double	6,2
5	33,8	Adjusted rhombic triacontahedron	Single	6,1
6	38,2	Dodecahedron	Single	5,2

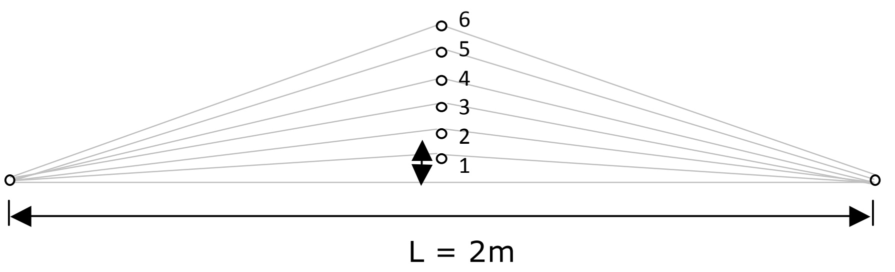


Figure 3.15: The ultimate geometric dimensions for the angulated part of the USC

3.2.3.2 Translational/polar part of the component

In comparison to the relative fixed geometry of the angulated elements in the domes, the polar and translational units allow a bigger freedom in geometry choice.

As mentioned in Chapter 2, a polar unit is simply obtained by moving the intermediate hinge of a plane translational unit away from the middle of the beams. This eccentricity of the revolute joint creates curvature when the units become deployed.

A beam from a translational and polar unit can simply be combined into one beam with several hinge positions. In this case the starting point of the geometrical methodology is a feasible eccentricity for the different hinge positions.

In the process of the search to feasible hinge positions, a good proportion of structural thickness is considered. Also, the fact that the scissor structure cannot be deployed too far must be taken into account. Figure 3.16 presents three polar arches of a barrel vault structure: the first structure is deployed too far, the second illustrates the maximum deployment to consider and the third shows a structure with a good deployment state. If a scissor mechanism is deployed too far, problems will rise during the transformation to a closed configuration. If a structure is forced into a state similar to that of the first structure in Figure 3.16, the phenomenon of snap-through must be taken into account. The maximum deployment must be in such a way that the lower semi-lengths of the scissor units are in each other's extension, as shown in the second structure of Figure 3.16.

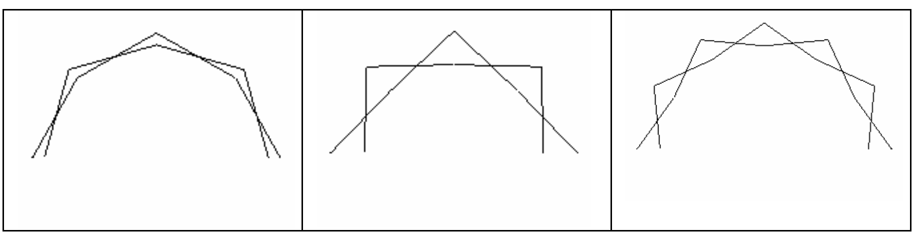


Figure 3.16: Three polar arches of a barrel vault structure in different deployment states

The length of the translational or polar beam is fixed on 2m as determined in the previous section. The parameters on which can be anticipated are the number of units and the eccentricity of the polar hinges. Configurations are investigated with a number of units from 4 to 16 to obtain a wide range of barrel vault spans and eccentricities are considered with a minimum distances of 5cm.

Appendix 2 gives an overview of the investigated barrel vaults. A polar hinge eccentricity of 5 cm can be used as from 7 units: less units result in an exceeding deployment. The same argument leads to the decision to apply eccentricity 10 cm for 5 and 6 units. An eccentricity of 16 cm is the following possibility to manage a barrel vault with 4 units. Hence, the conclusion is made to regard eccentricities of 5cm, 10cm and 16 cm. Figure 3.17 presents the ultimate geometric dimensions for the translational/polar part of the USC for which thirteen different barrel vault structures can be built geometrically.

	Hinge position [cm]	Number of polar units in arch of Barrel vault	Span Barrel vault [m]
1	5	U=7 U=8 U=9 U=10 U=11 U=12 U=13 U=14 U=15 U=16	8,3 9,4 10,5 11,6 12,6 13,6 14,6 15,5 16,4 17,3
2	10	U=5 U=6	5,6 6,5
3	16	U=4	4,1

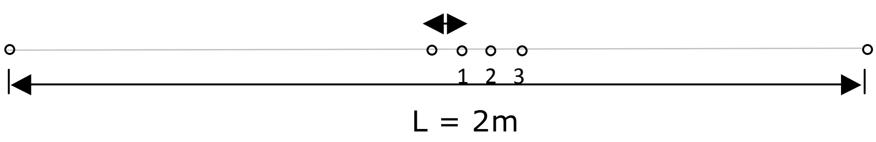


Figure 3.17: The ultimate geometric dimensions for the translational/polar part of the USC

3.2.4 Geometrical shape

In the previous section the geometrical dimensions were determined. The positions of the intermediate pivot hinges are found with the ability to compose them in such way that nineteen different architectural structures can be formed (Figure 3.18).

In this section the actual shape of the USC is determined. Two different options are considered to compose the hinges in a new type of scissor component. A first option is to connect the hinge positions with beams. An alternative is to form a type of stayed column with beams and pre-tensioned cables. The two options are illustrated in Figure 3.19.

With the combination of beams and pre-tensioned cables, the stayed beam could be more structurally efficient and could allow a reduction in weight. But there exists a higher complexity in terms of connections between the beams and cables. Also the applied pre-tension in the cables is a problem to solve in each component in the structure. Due to the large complexity of the stayed beam, the first option is chosen which implies a simpler manufacturing process.

Geometrically, the new Universal Scissor Component could be conceived like a truss (Figure 3.20). The middle strut divides the buckling length of the mast in two, which can be seen as an enlargement of the stiffness in one direction. But unlike in a real truss, in this element bending moments can exist due to the eccentricity of the hinge positions connecting the components to one another in a certain configuration.

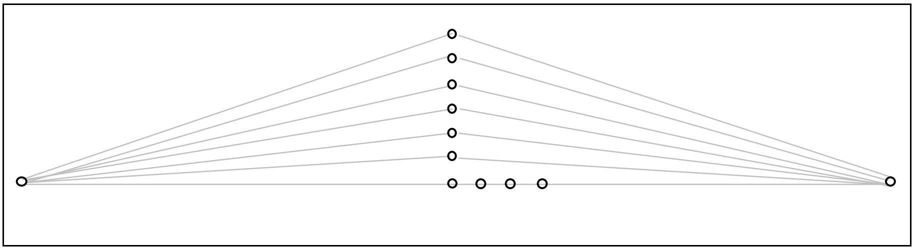


Figure 3.18: Geometrical positions of the hinges in the USC

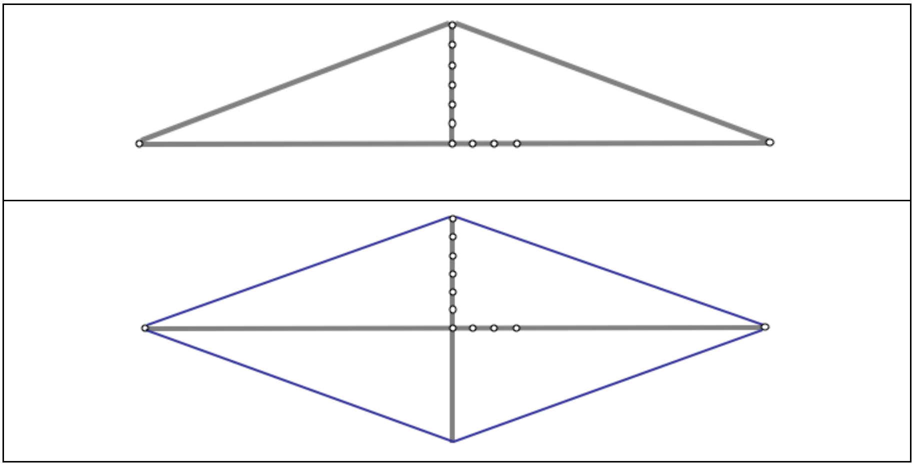


Figure 3.19: Option 1 and 2: formation of beams – stayed column (cables in blue)

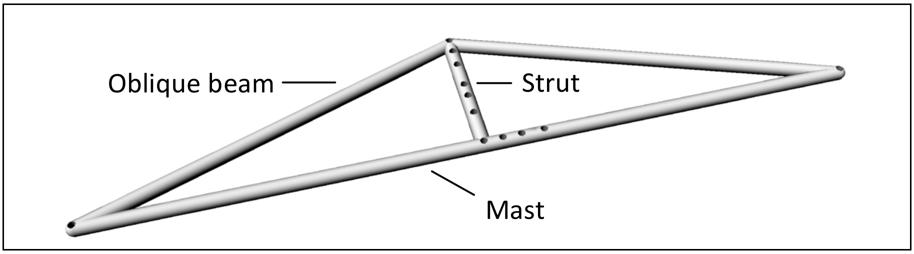


Figure 3.20: Designed Universal Scissor Component

3.2.5 From component to linkage

By way of illustration, the erection of a structure from a Universal Scissor Component to a linkage is presented. The following figures clarify the differences between barrel vaults and domes in the build-up. In this paragraph no implications involving technologic connections are considered. These are discussed in Chapter 5.

Figure 3.21 presents how a polar linkage is built from a USC by the example of the first hinge position at 5cm. Figure 3.22 considers a linkage using the second angulated hinge position at 16,4 cm.

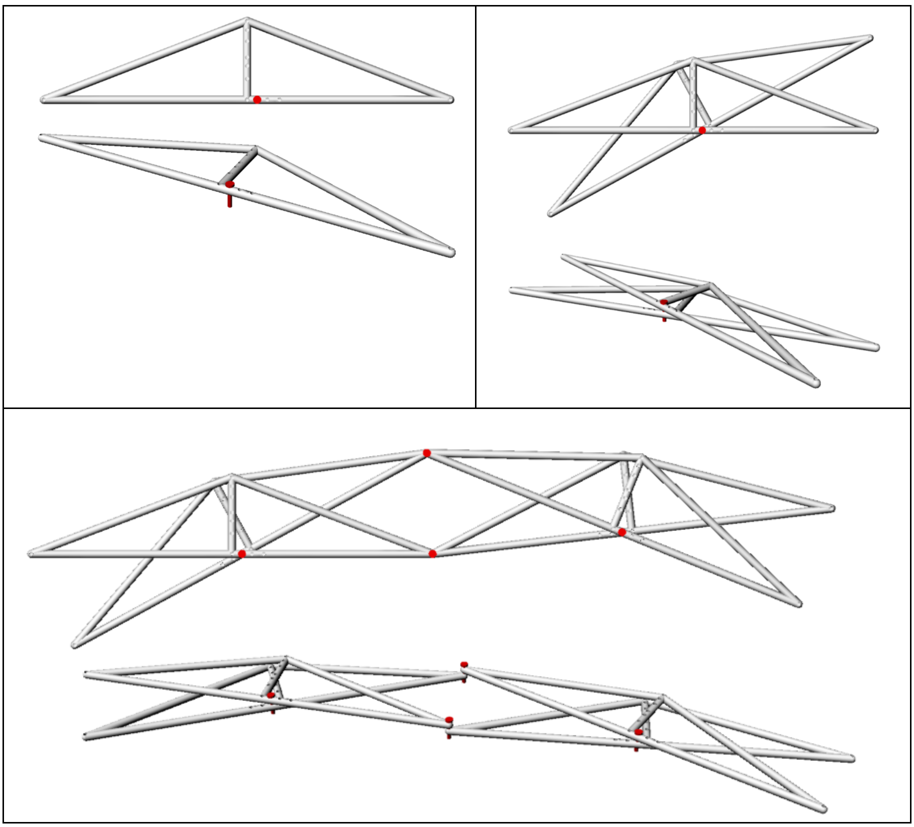


Figure 3.21: Top and perspective view: example of build-up from a USC to a scissor and further to a polar linkage for barrel vaults

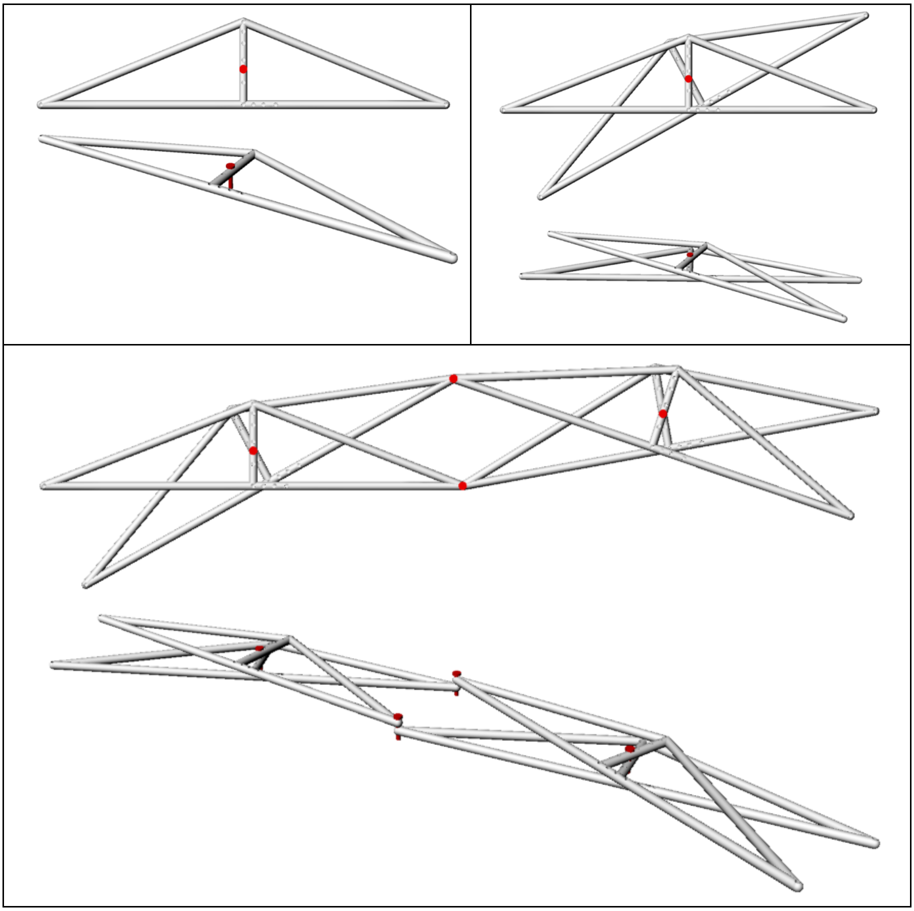
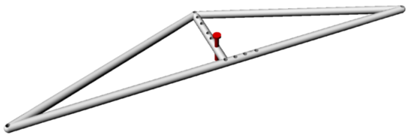
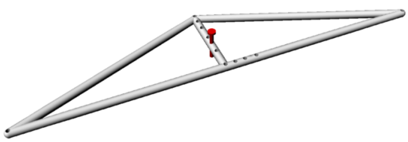
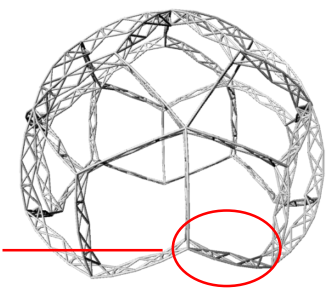
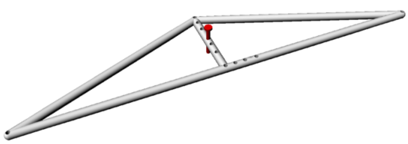
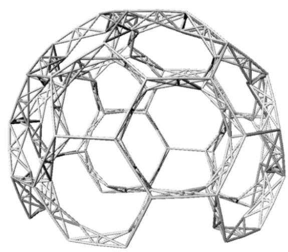


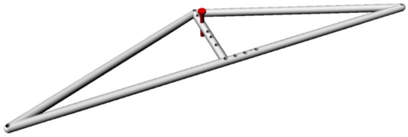
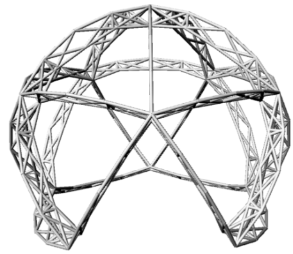
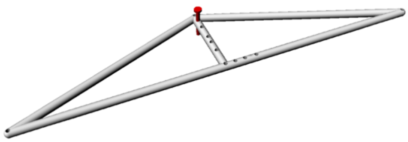
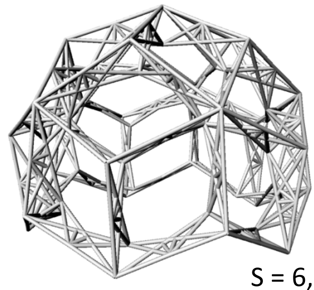
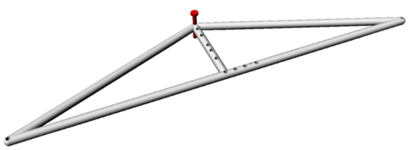
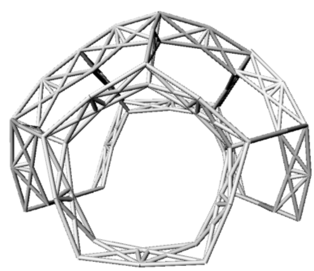
Figure 3.22: Top and perspective view: example of build-up from a USC to a scissor and further to an angulated linkage for domes

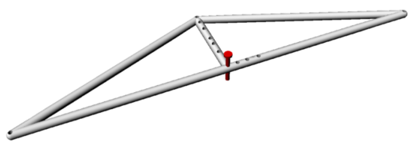
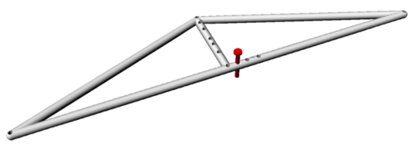
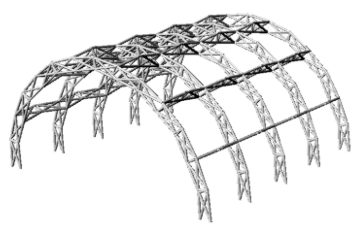
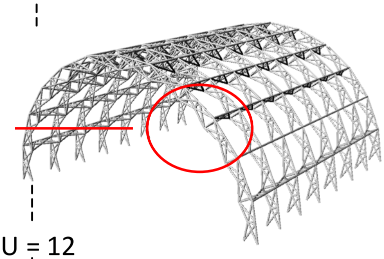
3.2.6 Overview

A total overview of the possibilities of the designed Universal Scissor Component is given. Figure 3.23 presents the different configurations of the structures by combining the USC's in a specific way. For each possible intermediate hinge position, which connects two components, the resulting domes and barrel vaults are given. Examples of detailed illustrations are given in the previous section.

The first part of Figure 3.23 shows the six different domes, each developed with a specific angulated hinge position. The type of polyhedron is mentioned as well as the issue whether the edge replacement involves a single or double unit. Also an indication about the span is given.
The second part presents the different cases for the barrel vaults. As explained in section 3.2.2.1, the translational units, with the intermediate hinge in the middle position, link the polar arches in the longitudinal direction. These are the same for each barrel vault. For the position at 5cm, ten barrel vaults can be composed going from 7 to 16 USC units, resulting in multiple spans. In case of the second eccentricity of 10cm, two barrel vaults are considered with 5 and 6 units. With the final hinge position of 16cm, the ability exists to generate a small barrel vault with 4 units and a span of 4,1m.

Domes	
USC and angulated intermediate hinge position	Structure configuration and span
	Buckyball – double unit per edge  S = 16,9m
 Details see Figure 3.22	Adjusted rhombic triacontahedron – double unit per edge  S = 12,2m
	Buckyball – single unit per edge  S = 8,5m

	Icosahedron – double unit per edge  S = 6,2m
	Adjusted rhombic triacontahedron – single unit per edge  S = 6,1m
	Dodecahedron – single unit per edge  S = 5,2m

Barrel Vaults	
USC and translational intermediate hinge position	Structure configuration and span
	For all barrel vault structures in longitudinal direction
USC and polar intermediate hinge position	Structure configuration, units and span
 Details see Figure 3.21	 U = 7 S = 8,3m  U = 12 S = 13,6m

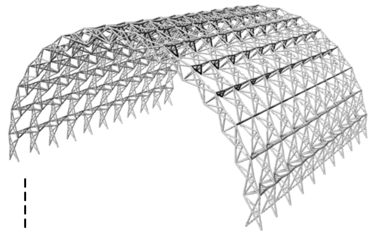
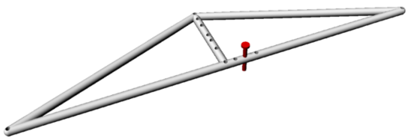
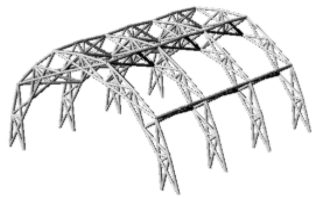
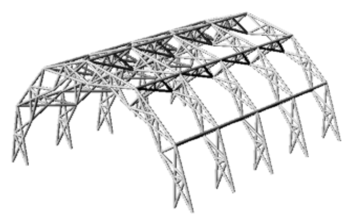
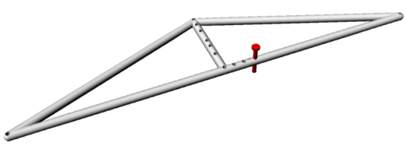
	 U = 16 S = 17,3m
	 U = 5 S = 5,6m  U = 6 S = 6,5m
	 U = 4 S = 4,1m

Figure 3.23: Total overview of the Universal Scissor Component and the structure configurations

3.3 Development: from mechanism to structure

In this section the matter of deployment and kinematic aspects is pursued, because the essence of deployable mechanisms is the deployment itself. More specifically, polar barrel vaults have a deployment behaviour which is different from the proposed angulated domes.

A two-dimensional scissor linkage has a single rotational degree of freedom (D.O.F) allowed by the intermediate hinge or revolute joint. When such linkages are placed on a grid (as described in paragraph 3.2.2), this rotational D.O.F. is, depending on the grid geometry, either preserved or removed. All this depends on the type of scissor units used (translational, polar, angulated), the type of grid they form (two-way, three-way, four-way) and the curvature (plane, single or double). An insight in the mobility of the mechanism is needed to understand to what degree constraints have to be added after deployment to turn it into a load bearing structure [De Temmerman, 2007].

3.3.1 Barrel vaults

As mentioned in section 3.2.2.1, the examined barrel vaults consist of polar arches connected by translational linkages. The geometrical construction of the polar mechanism is based on a circular arch and characterised by the parameters: Span (*S*), number of polar Units (*U*) and structural Thickness (*T*).

Ultimately, values for the semi-lengths of the beam l_i and l'_i and the deployment angle θ_{design} are obtained, which fully determine the geometry of the linkage in its deployed position. Furthermore, these three parameters suffice to study the deployment behaviour of the polar linkage. Now polar linkages have a peculiar property: during deployment, as the deployment angle θ increases from 0 (compact configuration) to θ_{design} (fully deployed configuration), the span S will not merely increase. As θ increases, a maximum span (S_{max}) is reached, only to decrease slightly until θ_{design} is reached (S_{design}) [De Temmerman, 2007]. This property is shown in Figure 3.24.

The deployment pattern of the polar linkage determines automatically the translational deployment generating the global transformation of the barrel vault. During the deployment, the length of the unit lines a decreases, which implies the deployment of the translational units. The length a and the fixed semi-lengths of the translational beams determine a unique θ for the translational units, linked to the θ of the polar scissors as illustrated in Figure 3.25. Using the cosine rule the relation can be written as:

$$a^2 + {k'_i}^2 + {l'_i}^2 - 2\, l'_i\, k'_i \cos\theta = (\, k_i + l'_i \,)^2 \qquad (3.1)$$

Figure 3.26 presents a barrel vault structure combined by USC's in four successive deployment positions.

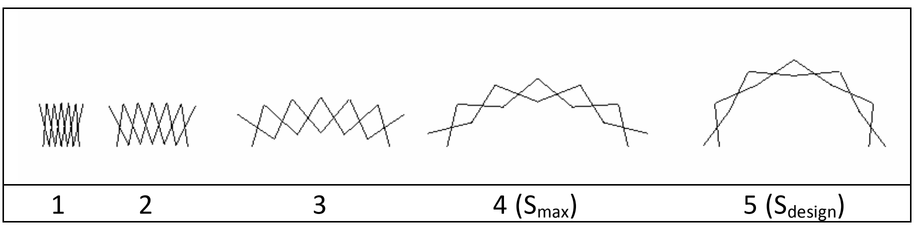


Figure 3.24: Deployment sequence of a polar linkage

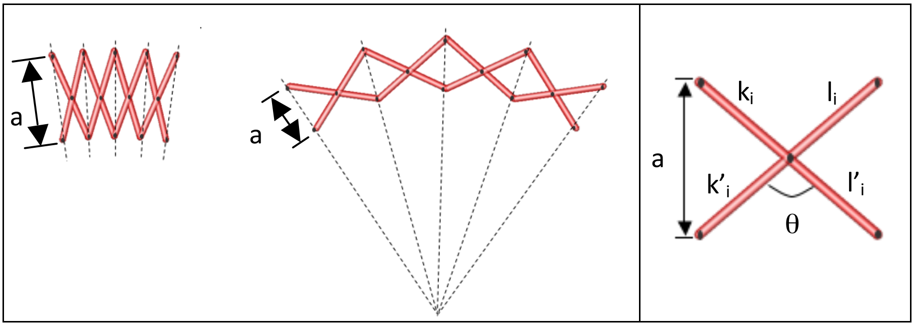


Figure 3.25: Illustration of the unique relation between the polar and translational deployment

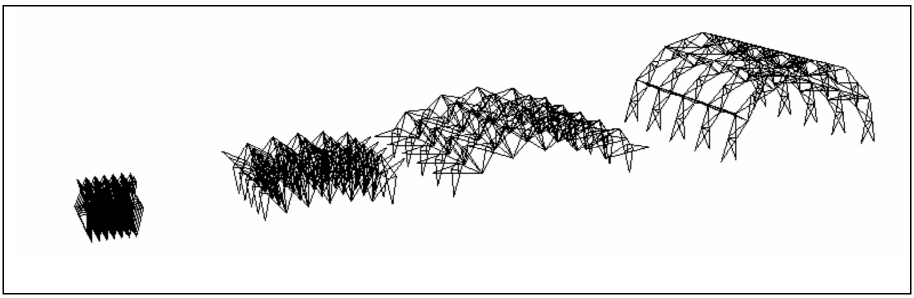


Figure 3.26: Barrel vault with USC's in four deployment configurations

De Temmerman [2007] proposed a hinged plate model to study the kinematic properties of a grid allowing to determine to what extent constraints have to be added to turn the mechanism into a structure. The model eliminates the rotational D.O.F. of the scissors, which is a minimum requirement for a scissor mechanism to act as a structure. In this case, after the elimination of the rotational D.O.F., it is deducted that the proposed barrel vault structure remains with no mobility. Therefore, there is no need for additional constraints, other than the one needed to eliminate the rotational D.O.F. of the original scissor mechanism. In practice however, it is suggested to fix all inner nodes touching the ground by pinned supports.

3.3.2 Domes

With angulated elements radially deploying closed loop structures can be created. This research focuses on deployable dome structures, capable of retracting to their own perimeter.

As explained in paragraph 3.2.2.2, the shape of the domes is based on a particular polyhedron. Angulated scissor units replace the edges of the polyhedron, while all the unit lines intersect in the centre point of the enclosing sphere of the polyhedron (Figure 3.27). The position of the unit lines stay fixed during deployment, ensuring that the overall geometrical shape does not change and only a variation in span is noticed.

The transformation of these closed loop structures is characterised by a translation on the unit lines of the end nodes of the scissor units. The deployment process is explained in Figure 3.28 by means of circles. The intermediate hinge of an angulated beam element is the centre point of the circle, while the end nodes determine the radius. As the circle centre point moves towards the sphere centre point, new intersection points with the unit lines are found. These intersection points are the new positions of the end nodes of the angulated element. If the lower intersection point coincides with the sphere centre point, the fully closed state is reached. The second angulated beam of the scissor unit is acquired by a mirror transformation with the bisector (blue dotted line) as mirror axis. The obtained angulated scissor unit is illustrated in Figure 3.12.

Figure 3.29 presents a deployable dome in three successive deployment positions.

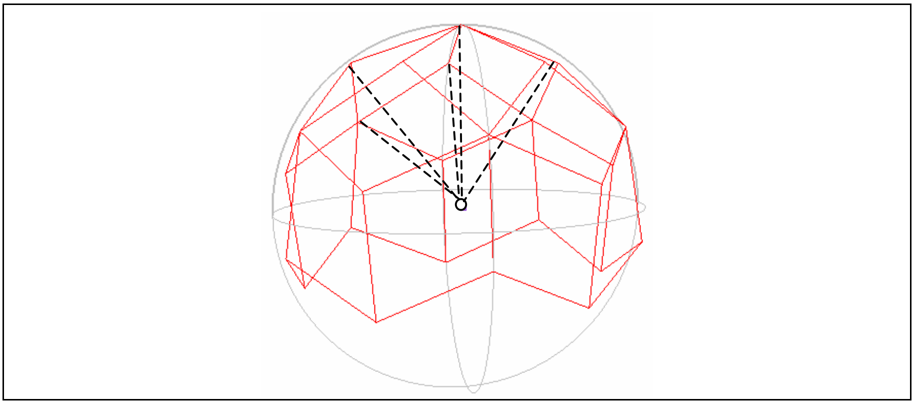


Figure 3.27: All the unit lines intersect in the centre point of the enclosing sphere

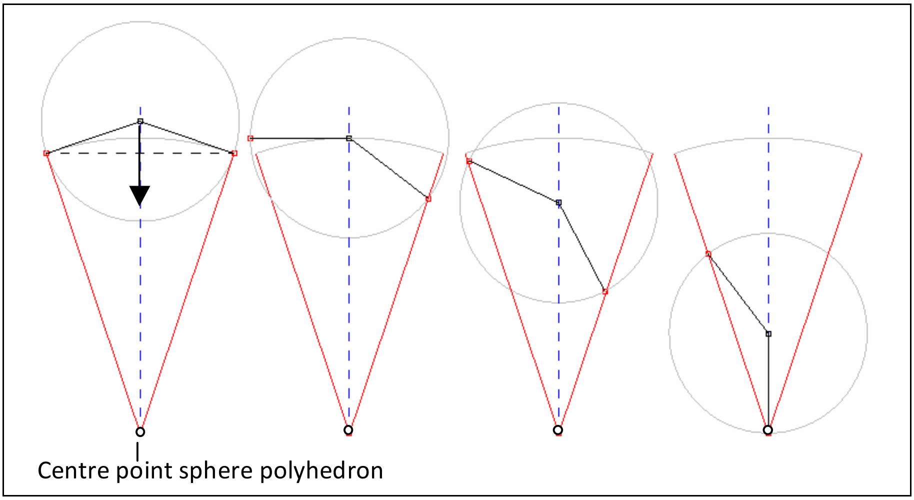


Figure 3.28: The deployment process of an angulated beam element

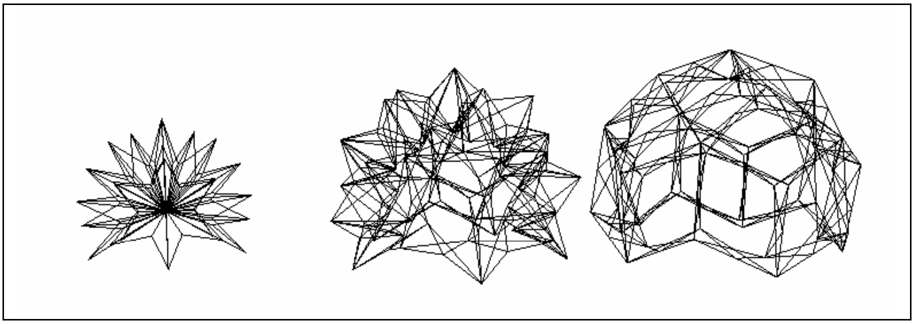


Figure 3.29: Deployable dome based on the adjusted rhombic triacontahedron from a fully closed to an open configuration

Also in case of the domes, the hinged plate model, proposed by De Temmerman [2007], can be used for a kinematic analysis. Analogous to the barrel vaults, the rotational D.O.F. of the scissor units is removed. After the removal of this D.O.F., the remaining mobility determines to what extent constraints have to be added.

Only in case of the icosahedron dome, the modules (grid cells) are triangulated, which means that there is no additional mobility and it is basically a single D.O.F.-mechanism. Therefore, it is sufficient to constrain the movement of the rotational degree of freedom of the angulated scissor units.

In case of the other proposed polyhedra domes, additional constraints have to be considered due to the non-triangulation of the grid cells. Cables or cross beams should be added to provide more stiffness and to exclude mobility in a module.

Ultimately, the lower nodes touching the ground are also fixed by pinned supports.

4 Structural Analysis of 2 case studies

4.1 Introduction

In the previous chapter, the principles behind the design of a Universal Scissor Component (USC) have been explained, resulting in a generic solution for deployable scissor structures. Based on different configurations of structures – barrel vaults and domes - decisions were made about the geometrical dimensions and the shape of the component. Considering feasible hinge positions, the design process resulted in a USC capable of configuring nineteen different architectural structures with specific deployment behaviour.

Besides the geometrical properties, a feasibility study is completed if the structural implications are also investigated. In this chapter a global configuration, consisting of the designed Universal Scissor Component (USC), is analysed structurally. Two situations should be considered when designing a deployable structure: the first during the deployment and the second in the service phase. Since in this research, during deployment no additional stresses are induced in the proposed foldable structures and the imposed loads and span are usually less than those for the fully deployed state, the structural part will concentrate on the analysis in the fully deployed configuration [Langbecker & Albermani, 2001].

In a larger construction the loads increase and the global instability becomes significant due to the scale-effect. Therefore, the calculation of the largest structure is assumed to result in a scissor component that satisfies the structural requirements in all system configurations. As mentioned before two types of configurations, each differing in multiple spans, can be built with the designed USC: domes and barrel vaults. Due to a preference for global instability in the longitudinal direction, which is not the case in dome configurations, a total detailed calculation is preformed on the largest barrel vault.

The result of my research is a first feasibility study of the designed structures; hence focus is put on those points which are assumed to be the most critical and a design methodology is offered enabling to cover the remaining cases. It is explained how a simplified approach is used for the preliminary design in SCIA ESA-PT using the Eurocode standards.

In a first part the global geometry of the analysed structure is described. The outline of the structural model and the assumptions made are then discussed. In the next part the climate loads, wind and snow, are calculated implementing Eurocode 1 [2006]. Finally, two case studies of the barrel vault configuration are examined structurally: (1) the global structure and (2) the global structure with passive and active cable segments.

4.2 Geometry

In the scope of a first type of calculation in this research, the analysis is focussed on the largest barrel vault, which is assumed to be representative for all the remaining configurations. The geometry of the global barrel vault analysed in the two following case studies is presented in Figure 4.1. The semi-cylindrical shape consists of 11 arches formed by the USC composed in a polar configuration, which are linked together with translation USC's creating a deployable structure. Using polar scissor arches creates curvature in the transverse direction of a quadrangular scissor grid. In combination with longitudinal translational units they form relatively common and simple barrel vault structures.

The geometry is based on a semicircle with a radius of 8,6m. It has a span of approximate 17m and the structural thickness is less than 1m. The barrel vault is an open structure with neither back nor front and encloses a quite large architectural area of 308 m^2.

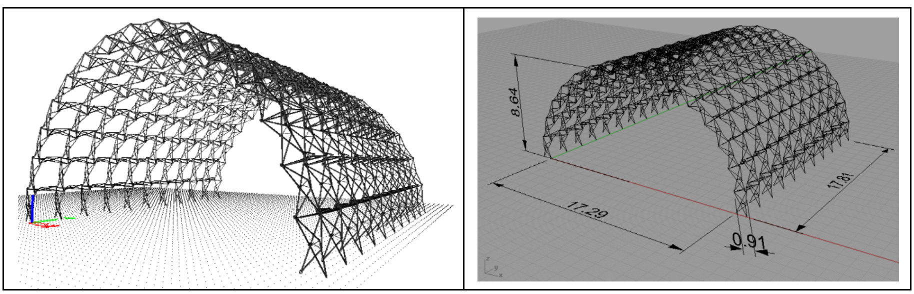


Figure 4.1: The global geometry of the barrel vault and some dimensions

4.3 Structural model

The structural analysis is done using the software SCIA ESA-PT [SCIA, 2007]. It is a software tool to design, calculate and evaluate constructions conform the applying building standards.

4.3.1 Introducing dummies

SCIA ESA-PT is generally aimed at standard constructions. Therefore model assumptions must be made to investigate deployable scissor structures. The revolute joint allowing a relative rotation between the beams cannot be simulated in SCIA ESA-PT.
The solution is given by introducing "dummies". These are small beams, offsetting the USC's in different planes and providing the possibility to define correct releases between the beams (Figure 4.2). The dummies are a good approximation of the real life structure: due to the thickness of the sections of the beams these cannot be connected in their theoretical intersection nodes. This requires the realisation of specific joints (cfr. Chapter 5).

The dummies are modelled as steel beams with a large stiffness (E= 1e6 MPa) and moment of inertia to make their influence on the structure design as small as possible. The dummies (or the joints) are not analysed and dimensioned in this dissertation. Therefore, multiple aspects should be considered such as structural tolerances, imperfections and friction. These could be investigated in a more profound research.

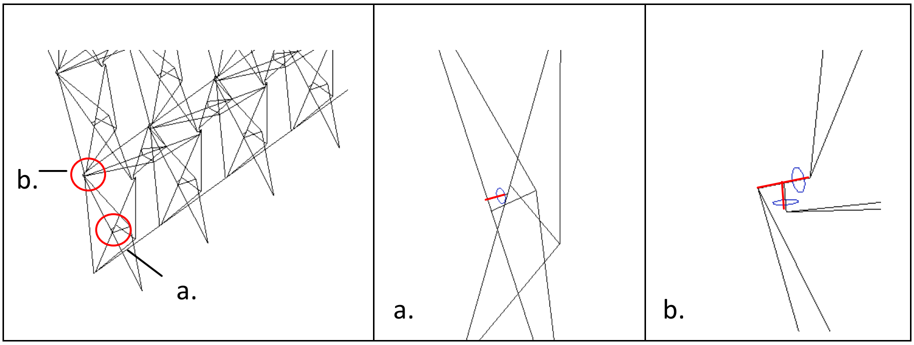


Figure 4.2: The beamlike dummies (red): blue circles indicate the possible directions of rotation

4.3.2 SLE beams

As explained in Chapter 3 the USC consists of three types of beams, each with its own function in the geometrical design: (1) the mast (*L*= 2m), (2) the oblique beams (*L*= 1,07m) and (3) the strut (*L*= 0,38m). These are modelled as type S235 steel and are given a profile section MSRR[4]. In this initial study, where the implications of the geometry are examined structurally, steel is chosen as material for a first design calculation. A further research could explore the effects of aluminium or even composites as material.

The beams are connected with each other in hinged nodes. If the component were realised, welding connections would be used, which are better simulated by hinged nodes instead of clamped connections.

[4] MSRR is a profile of the European library in SCIA ESA-PT: Structural hollow section/ Vallourec-Mannesmann Tubes/Ed.1998.

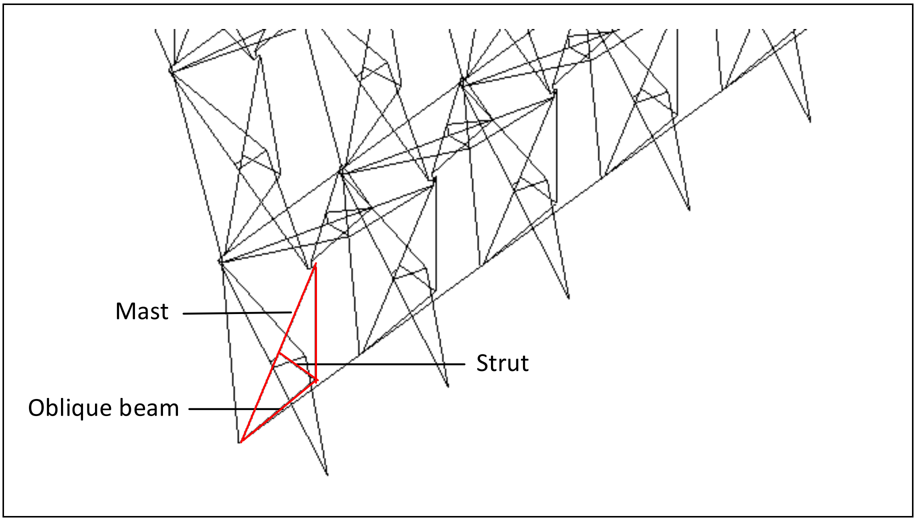


Figure 4.3: Detail view of a universal scissor component in the global structure (red)

4.3.3 Membrane as plates

Generally, mobile deployable structures consist of a weather protecting membrane. The software EASY provides the tools to analyze such membranes under loading conditions, defining the pre-tension in membrane and cables.
This thesis however focuses on the feasibility of the designed steel scissor structure, without details regarding a more accurate load transfer with a pre-tensioned membrane. Therefore 2D plate elements are introduced in the structure to form a 3D shell. In this case the calculated wind and snow loads can be easily generated as area forces in SCIA ESA-PT.

The 2D elements are modelled as thin aluminium plates (thickness 2mm) with an adjusted self weight to approach the self-weight of a membrane in case of mobile structures (600gr/m^2). In contrast to membranes, which are characterized by a plane stress state, the aluminium plates have a certain bending stiffness out of plane. This can be neglected according to the small plate thickness, which make these plates a good approximation of a membrane model for a preliminary design.

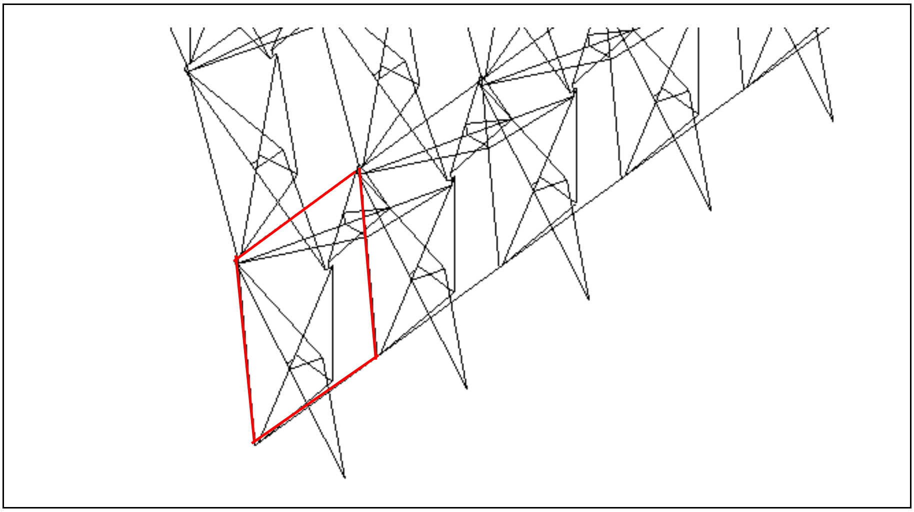


Figure 4.4: Detail view of plate element in the global structure (red)

4.3.4 Cable segments

In case study 2 passive and active cable segments are added to the global barrel vault structure. An active cable runs through the mechanism, connecting upper and lower nodes along its path. After deployment it is locked to stiffen the structure. Passive cables are upper cables connecting the upper nodes and lower cables are connecting the lower nodes. As the active cable fulfils the deployment, the passive cables become tensioned, adding an extra stiffness to the structure and avoiding that the structure deploys too far (Figure 4.5).

In SCIA ESA-PT it is not possible to simulate a full-length cable passing through the structure. Therefore the passive and active cables are modelled in discrete cable segments. Moreover no pre-tension is introduced in the cables, which provides in reality a contribution to the overall structural performance. An iterative process would determine the optimal pre-tension in the cables, which falls outside the scope of this work.
The cable segments are modelled as steel RD[5] profiles, that can only transfer tension forces.

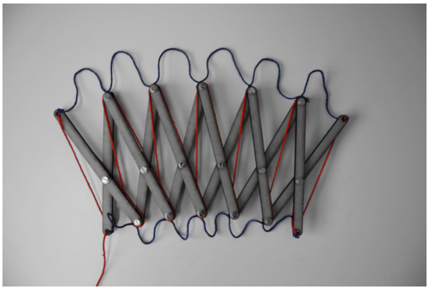
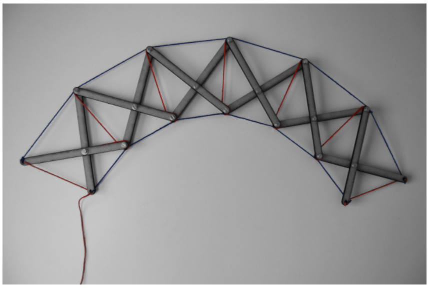
	
Undeployed state: active and passive cables are loose	Fully deployed state: active cable is shortened and locked and the passive cables are at full tension

Figure 4.5: Interaction between active (red) and passive cables during deployment

[5] RD is a profile of the European library in SCIA ESA-PT: round section beam/ Stahl im Hochbau/14 Auflage Band I/Teil 1.

4.3.5 Global model

The lower and inner nodes touching the ground are fixed with pinned supports (only the rotation is held free). This removes the mobility from the mechanism and enables it to act as construction and transfer loads.
The global coordinate system and the local coordinate system of a beam are shown in Figure 4.6.

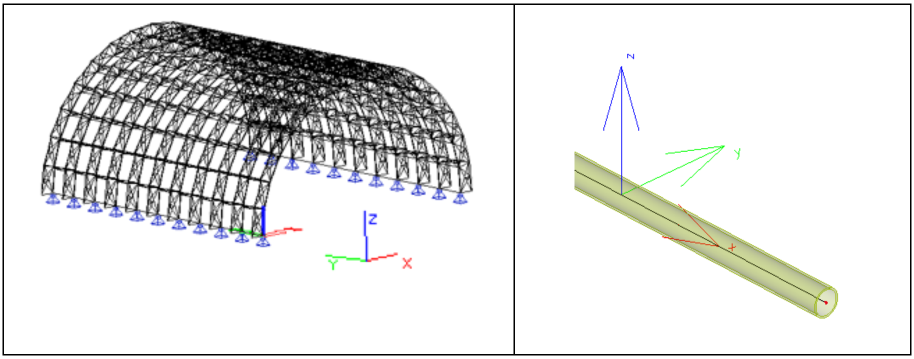


Figure 4.6: Global coordinate system (left) and local coordinate system (right)

4.4 Load cases

The barrel vault structure is dimensioned with the action of climate loads and self-weight. Three different climate loads are considered: transverse wind, longitudinal wind and snow (Figure 4.7). In a more profound analysis other wind directions should also be examined. In a future edition of SCIA ESA-PT the ability will exist to generate automatically a 3D wind on the structure, investigating all critical wind directions. Not possible to use the 3D wind generator on this open scissor construction, only two wind directions are used for this simplified approach. The load cases are calculated as prescribed in Eurocode 1 [2006]. Whenever assumptions are made or simplifications are done, the most unfavourable value is chosen.

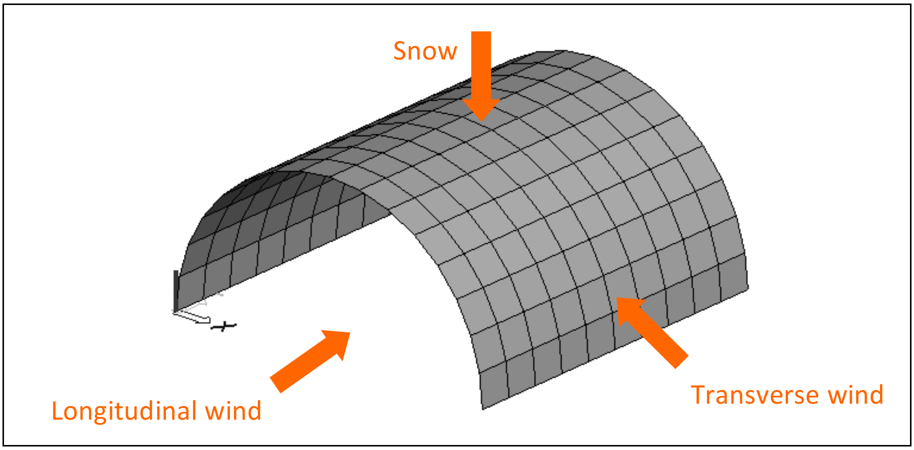


Figure 4.7: Schematic representation of climate loads

4.4.1 Wind

The reference wind pressure is expressed as:

$$q_{\mathrm{ref}} = \rho/2 \,.\, v^{2}_{ref} \tag{4.1}$$

$$v_{ref} = c_{DIR} \,.\, c_{TEM} \,.\, c_{ALT} \,.\, v_{ref,0} \tag{4.2}$$

The determining factors are presented in Table 4.1 . This results in a value for q_{ref} = 0,279 kN/m^2.

The total wind pressure acting on the surfaces (2D plates) is obtained from:

$$w = w_e - w_i \tag{4.3}$$

$$w_e = q_{ref} \,.\, C_e(z_e) \,.\, C_{pe} \tag{4.4}$$

$$w_i = q_{ref} \,.\, C_e(z_i) \,.\, C_{pi} \tag{4.5}$$

The pressure on the external and internal surfaces is determined by a pressure coefficient related to the geometry of the structure, which means the surface is divided into several zones, each with their own C_p.

Eurocode 1 [2006] prescribes how the pressure coefficients for a barrel vault structure can be determined in case of transverse wind. For the situation of longitudinal wind, assumptions must be made by approximations to other structure geometries. In this latter case the pressure coefficients are obtained by referring to [De Temmerman, 2007].

Table 4.2 gives a summary of all obtained values for the external and internal wind pressure and exposure coefficients and the resulting total pressure per load zone and wind direction. Figure 4.8 shows the different load zones and presents the wind pressure and suction.

Table 4.1: Factors determining the reference wind pressure

ρ	Air density (kg/m^3)	1,25
c_{DIR}	Direction factor	1
c_{TEM}	Temporary factor (one month exposure – November)	0,806
c_{ALT}	Altitude factor	1
$v_{ref,0}$	Basic velocity (m/s)	26,2

Table 4.2: Factors determining the total wind pressure

z_e	External reference height (m)				4,325
$C_e(z_e)$	Exposure coefficient				1,845
$C_r(z_e)$	Roughness factor				0,847
k_t	Terrain factor – terrain factor II				0,19
z_i	Internal reference height (m)				4,325
μ	Opening ratio				1
Transverse wind					
Zone	C_{pe}	w_e (kN/m^2)	C_{pi}	w_i (kN/m2)	w (kN/m^2)
A	0,8	0,411	-0,5	-0,257	0,668
B	-1,2	-0,617	-0,5	-0,257	-0,360
C	-0,4	-0,304	-0,5	-0,257	-0,047
Longitudinal wind					
Zone	C_{pe}	w_e (kN/m^2)	C_{pi}	w_i (kN/m2)	w (kN/m^2)
D	-0,6	-0,309	0,14	0,072	-0,381
E	-0,3	-0,154	0,14	0,072	-0,226
F	-0,2	-0,103	0,14	0,072	-0,175

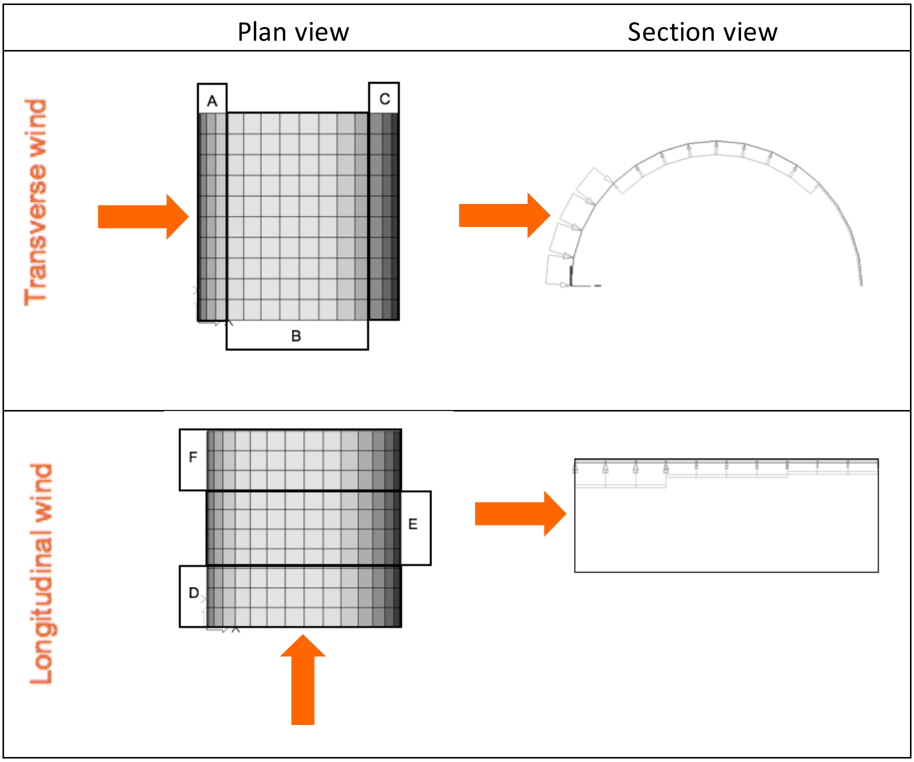


Figure 4.8: Schematic representation of wind loads and zones

4.4.2 Snow

The snow load is calculated with the following formula:

$$s = \mu_i \cdot C_e \cdot C_t \cdot s_k \tag{4.6}$$

The determining snow factors are presented in Table 4.3, resulting in a snow load of 1kN/m^2. Because surfaces with an inclination of 60° and above are assumed to be without snow, the snow load is only applied to the zones at the top of the canopy, as shown in Figure 4.9.

Table 4.3: Factors determining the snow load

μ_i	Form factor	2
C_e	Exposure coefficient	1
C_t	Temperature coefficient	1
s_k	Characteristic snow load on the ground (kN/m^2) for a transportable accommodation and snow accumulation within 24 hours [STANAG 2895, 1990]	0,5

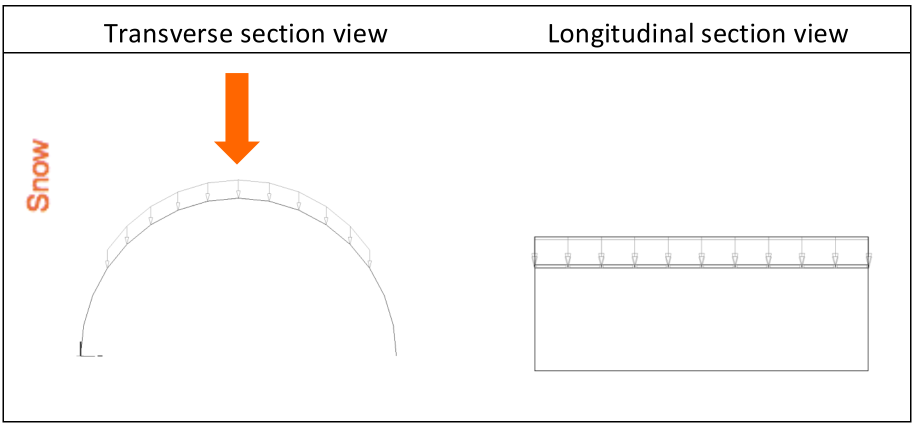


Figure 4.9: Schematic representation of snow load

4.4.3 Load combinations

Considering the three mobile loads – transverse wind, longitudinal wind and snow – seven load combinations are compiled, according to the prescribed combination factors (Table 4.4). The two wind loads have a distinct direction; hence it is assumed that these cannot interact together.

With G the permanent loads, Q the mobile loads, γ their respective safety factors and X a factor in case of additional mobile loads, the total load can be written in its general form:

$$F = \Sigma_i \gamma_G G_{k,i} + \gamma_Q Q_{k,1} + \Sigma_j \gamma_Q X_{0,1} Q_{k,j} \quad (4.7)$$

In the ultimate limit state (ULS) the strength (determination of sections) and stability (buckling analysis) of the structure are verified, while the service limit state (SLS) is used for the control of the stiffness (displacements). The ULS and SLS load cases are used to determine the actions on the beam structure by loading of the plate elements.

Table 4.4: Combination factors and load cases

	γ_G	γ_Q	$X_{0,1}$ wind	$X_{0,1}$ snow
ULS	1,35	1,5	0,6	0,5
SLS	1	1	0,6	0,5
Load case	Permanent load	Main mobile load	Additional mobile load	
ULS/SLS 1	Self-weight	Snow		
ULS/SLS 2	Self-weight	Transverse wind		
ULS/SLS 3	Self-weight	Longitudinal		
ULS/SLS 4	Self-weight	Snow	Transverse wind	
ULS/SLS 5	Self-weight	Transverse wind	Snow	
ULS/SLS 6	Self-weight	Snow	Longitudinal wind	
ULS/SLS 7	Self-weight	Longitudinal	Snow	

4.5 Structural analysis

4.5.1 General approach

According to Eurocode 3 [2007], a first order (linear) analysis may be used for a structure, if the increase of the relevant internal forces or moments or any other change in structural behaviour caused by deformation can be neglected. This condition may be assumed to be fulfilled, if the following criterion is satisfied:

$$\alpha_{cr} = F_{cr}/F_{Ed} \geq 10 \tag{4.8}$$

α_{cr} is the factor by which the design loading has to be increased to cause elastic instability in a global mode. If α_{cr} has a value lower than 10, a second order (non-linear) calculation needs to be executed. A structural analysis is performed in ULS to examine the structure's strength (section design) and local and global stability (buckling control). SCIA ESA-PT works in accordance with Eurocode 3 [2007].

Each type of SLE beam (mast, oblique beam and strut) is optimised separately. The structure design is performed by means of an iterative analysis with the area of the section (thus minimum weight) as optimisation criterion. After an optimisation a following calculation is necessary to redistribute the internal forces, reflecting the right results of the optimisation. Each time different optimisation steps can distinguish other sections as optimal. The recurrence of optimisation followed by a calculation can lead to an infinite cycle.
The optimisation is characterised by the factor f:

$$f = \sigma_{Ed} / \sigma_{max} \tag{4.9}$$

The factor f may not exceed the value of 1, otherwise the section will fail under the design load.

A buckling analysis is performed automatically according to the calculated buckling parameters and the global stability is checked based on a calculation with a stability load combination.
When assigning sections to the beams the following condition about the ratio between the thickness (*t*) and the diameter (*d*) is taking into account:

$$0{,}02 < t/d < 0{,}15 \tag{4.10}$$

The lower value is the minimum bound preventing local buckling phenomena [Eurocode 3, 2007]. The upper bounder is set as commercially available; larger ratios are less structural efficient [Latteur, 2000].

The stiffness of the structure is analysed under SLS by checking the displacements of the nodes. Because this is a concept for a mobile structure, the same strict requirements as for permanent buildings do not apply.

The internal forces in the beams are expressed in the local coordinate system, while the reaction forces and the displacements are presented according to the global coordinate system.

4.5.2 Case study 1

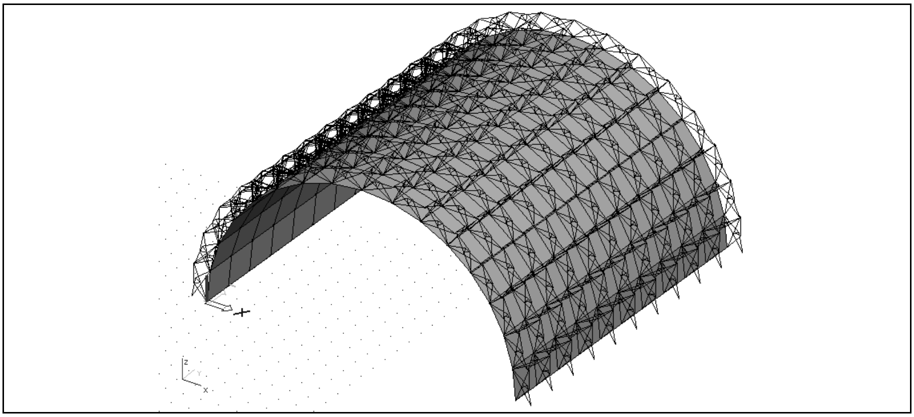


Figure 4.10: Structure analysed in case study 1: barrel vault with 2D plate elements

The main goal of this chapter is to dimension the designed structure under the prescribed climate loads and to investigate under a simplified approach whether this dimensioned structure is feasible to construct. In this first case study the global barrel vault is analysed (Figure 4.10).

For this structure, α_{cr} has a value lower than 10, which means that a non-linear calculation has to be executed, as mentioned in section 4.5.1. The fact that α_{cr} is larger than 1 means that the structure can globally carry the design loads: the global stability is ensured.
Table 4.5 summarises the results for case study 1. The different detailed calculation notes are given in Appendix 3.

The governing load combinations for the design of the sections are non-linear ultimate limit state NLULS 1 (self-weight + snow) and NLULS 4 (self-weight + snow + transverse wind). Figure 4.11 shows how the structure is deformed in both cases.

The obtained maximal section diameter is approximate 5 cm, which can be considered as a feasible section compared to the enclosing area of 308 m^2. The total weight of the structure is 10300 kg, or 33,5 kg/m^2. Because the joints are not structurally analyzed and designed, their weight is not taken into account. On the contrary, a standard weight for the membrane of 600 gr/m^2 is incorporated.
In this case the weight of one Universal Scissor Component is 15,4 kg. This value is acceptable for handling of charges manually by workers as described by the legal codex concerning well-being at work [ARAB, 2002][6].

It can be noticed that the bending moments are very small, near zero. This means that the structure with the designed scissor component behaves structurally efficient, minimizing bending moments.
The displacements values (max 35mm) seem acceptable for this type of structure and these will not degrade the serviceability of the structure.

[6] Although the maximal weight of a load, manually handled, by a worker is not legally determined, an acceptable indicatory value is set at 23 kg [ARAB, 2002].

Table 4.5: Numerical results of case study 1

α_{cr}	Critical loading coefficient	5,06 → Non-linear
S235	Steel material	-
W_{tot}	Total weight [kg]	10301,1
w_a	Weight/area [kg/m2]	33,5
w_{USC}	Weight of one Universal Scissor Component [kg]	15,4

Strength		
Section [mm]	f	Load combination (LC)
Mast 51.0x2.9mm	0,77	NLULS 4
Oblique beam 51.0x3.2mm	0,97	NLULS 1
Strut 21.3x2.3mm	0,73	NLULS 4

Internal forces				
Section [mm]	N_{Ed} [kN]	$M_{y,Ed}$ [kNm]	$M_{z,Ed}$ [kNm]	LC
Mast 51.0x2.9mm	-12,63	0,00	0,16	NLULS 4
Oblique beam 51.0x3.2mm	-24,30	0,21	0,25	NLULS 1
Strut 21.3x2.3mm	-8,46	0,00	0,10	NLULS 4

Reactions					
R_x [kN]	LC	R_y [kN]	LC	R_z [kN]	LC
-13,37	NLULS 4	-32,07	NLULS 4	62,76	NLULS 4

Displacements					
U_x [mm]	LC	U_y [mm]	LC	U_z [mm]	LC
34,00	NLSLS 2	-30,20	NLSLS 2	-34,70	NLSLS 2

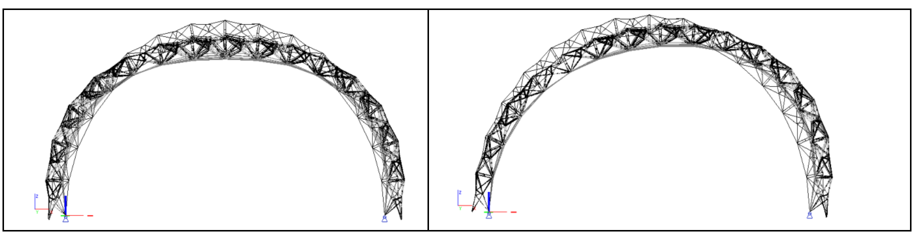


Figure 4.11: Front view of the deformed structure under NLULS 1 (left) and NLULS 4 (right) (no scale)

4.5.3 Case study 2

This second case study will examine the effects of introducing passive and active cable segments, which in reality contribute to the deployment and the overall structural performance. As mentioned in the description of the structural model (paragraph 4.3.4), no pre-tension force is applied in the cables. Figure 4.12 and 4.13 present the global structure with cables and some details.

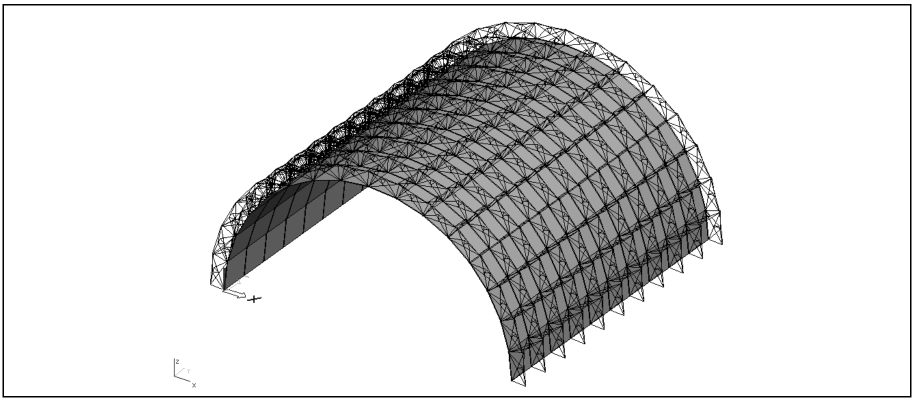


Figure 4.12: Structure analysed in case study 2: barrel vault with 2D plate elements and cable segments

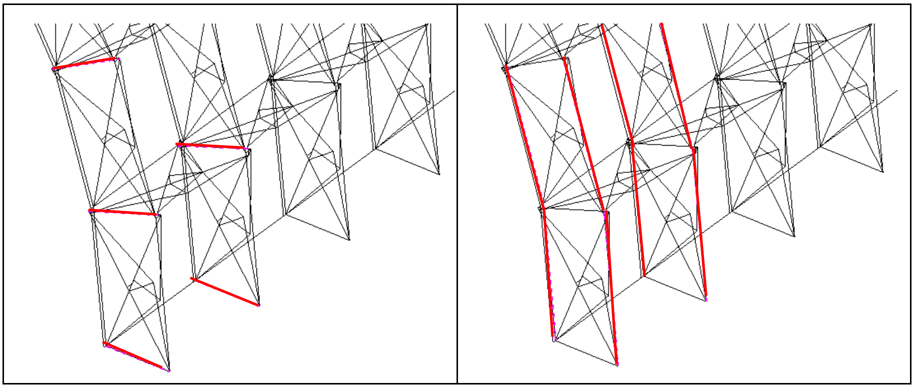


Figure 4.13: Detail view of the active (left) and passive (right) cable segments (red)

For this structure, α_{cr} has also a value lower than 10, which means that a non-linear calculation has to be executed.
Table 4.6 summarises the results for case study 2. The different detailed calculation notes are given in Appendix 4.

The governing load combinations for the design of the sections are NLULS 2 (self-weight + transverse wind) and NLULS 4 (self-weight + snow + transverse wind). Figure 4.14 shows how the structure is deformed in both cases.

The maximal section diameter obtained is approximate 4,2 cm. The total weight of the structure is 9025 kg, or 29 kg/m^2, including standard membrane and cable weight. Introducing cable segments has a positive effect on both the total weight and the section.
The weight of a single USC is now decreased to 12,5 kg, which excludes manual lifting problems for workers [ARAB, 2002].

When considering Table 4.5 (without cables) and Table 4.6 (with cables), a noticeable difference is observed in the reaction forces and the displacements. The effect of incorporating a cable is an alteration of the stiffness, resulting in a redistribution of forces and a decrease in peak displacement values. It can be considered as a self-stabilised structure where the forces are dealt with internally. The structure with cables has a lower total weight and results in lower reaction forces, which mean a smaller impact on the site (lightweight anchors). This is an important aspect focussing on the durability of mobile constructions.

Table 4.6: Numerical results of case study 2

α_{cr}	Critical loading coefficient	5,93 → Non-linear
S235	Steel material	-
W_{tot}	Total weight [kg]	9024,6
w_a	Weight/area [kg/m2]	29,3
w_{USC}	Weight of one Universal Scissor Component [kg]	12,5

Strength		
Section [mm]	f	Load combination (LC)
Mast 42.4x2.6mm	0,85	NLULS 2
Oblique beam 33.7x4.5mm	0,91	NLULS 4
Strut 21.3x2.3mm	0,50	NLULS 4
Passive cable d12mm	0,90	NLULS 4
Active cable d8mm	0,89	NLULS 2

Internal forces				
Section [mm]	N_{Ed} [kN]	$M_{y,Ed}$ [kNm]	$M_{z,Ed}$ [kNm]	LC
Mast 42.4x2.6mm	-20,28	0,00	0,29	NLULS 2
Oblique beam 33.7x4.5mm	-20,56	0,15	-0,01	NLULS 4
Strut 21.3x2.3mm	-5,02	0,00	-0,03	NLULS 4
Passive cable d12mm	23,84	-	-	NLULS 4
Active cable d8mm	10,48	-	-	NLULS 2

Reactions					
R_x [kN]	LC	R_y [kN]	LC	R_z [kN]	LC
-13,86	NLULS 4	-24,96	NLULS 1	45,11	NLULS 1

Displacements					
U_x [mm]	LC	U_y [mm]	LC	U_z [mm]	LC
27,10	NLSLS 2	-25,10	NLSLS 2	-27,80	NLSLS 2

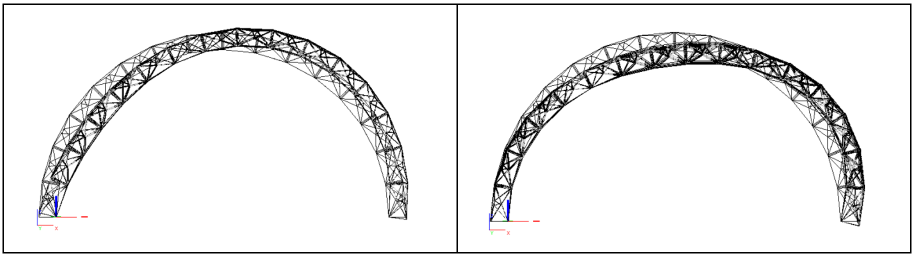


Figure 4.14: Front view of the deformed structure under NLULS 2 (left) and NLULS 4 (right) (no scale)

5 Technologies

5.1 Joints

Crucial to any deployable structure are the joints. Every unit, whether it is a standard scissor unit or composed by the Universal Scissor Component, consists of a revolute joint (intermediate hinge) characterised by a rotational degree of freedom, allowing deployment into a compact bundle. At the end nodes of the scissor units, the beam elements are connected by another revolute joint.

In reality, the members and the joints have discrete dimensions, unlike the theoretical geometric line models which have zero thickness. In theory, both components of a unit lie in a common plane. As opposed to the theoretical one-dimensional coplanar scissors, the physical beams are not in the same plane. A scissor unit has an imaginary centre plane, which separates the two components lying on either side of that plane (Figure 5.1) [De Temmerman, 2007].

The size of the joint (or hub) is influenced by the beams it has to connect. The wider the section and the higher the number of beams coming together in one joint, the larger the radius of the joint must become, in order to accommodate all elements without interference during deployment.

The sections of the beams are indeed an important aspect to consider if a large, unpractical joint is to be avoided. As a result of the structural analysis in the first case study, as described in the previous chapter, a maximum section of 51mm has been found. This dimension can result in a feasible joint, which does not have to take on large proportions. The second case study has proven that cable segments have a positive effect leading to a maximum section of 42,4mm, and thus resulting in an even smaller hub.

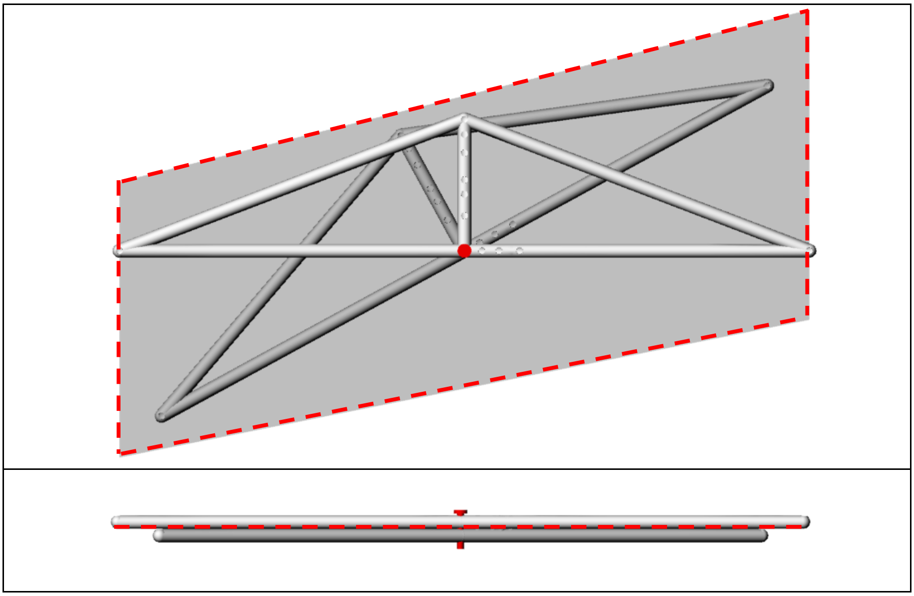


Figure 5.1: Front and top view of a USC unit relative to the theoretical plane

Escrig [1985] proposed some constructional aspects, which form the basis of the whole design of a feasible deployable structure. Some of his particular solutions for joints are shown in Figure 5.2. It is important to realise that the hubs will have to allow every possible movement to ensure a stress-free deployment.

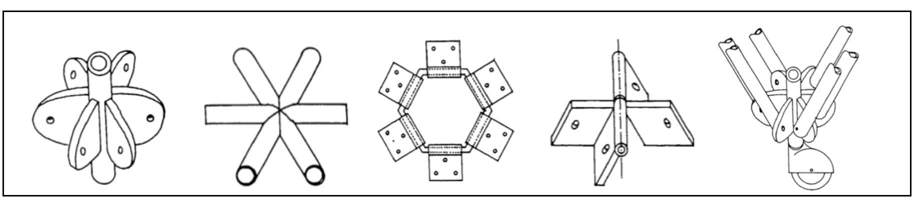


Figure 5.2: Some examples of joint proposed by Escrig [1985]

In case of the investigated barrel vault structures, maximally four beams are connected in one joint: two polar and two translational components (see section 3.2.2.1). A suggestion for a possible joint is illustrated in Figure 5.3. As shown in the latter figure, the beams can be provided with wedge-shaped end pieces, which allow them to be as compactly arranged as possible resulting in an even smaller joint.

In case of the six domes as presented in paragraph 3.2.2.2, either three or five beam components come together in all the vertices, depending on the polyhedron. Therefore, two types of hubs are distinguished. The joints can have the same shape as the one for the barrel vault structures, but one joint consists of three fins and for the second dome hub five fins are necessary (Figure 5.4).
It can be concluded that for the nineteen deployable scissor structures, combined by a single Universal Scissor Component, only three different types of joint are required.

The centrelines of all units connected by a certain joint have a single intersection point P, as Figure 5.5 presents. In the deployed position, this intersection point P lies on the vertical axis through the joint. As the structure is compacted towards its undeployed state, point P moves further upward until all centrelines are parallel.
The beams are connected to the fins in a hinge point which is not at point P. This eccentricity has an effect on the structural performance of the structure in the sense that this geometric imperfection induces a second order effect, i.e. bending in the beams [De Temmerman, 2007].

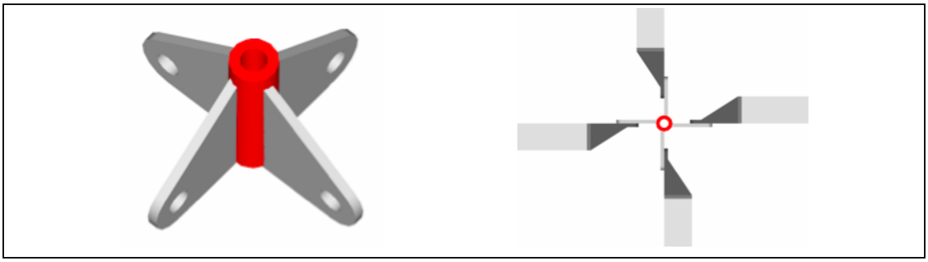


Figure 5.3: Joint connecting four beam components [De Temmerman, 2007]

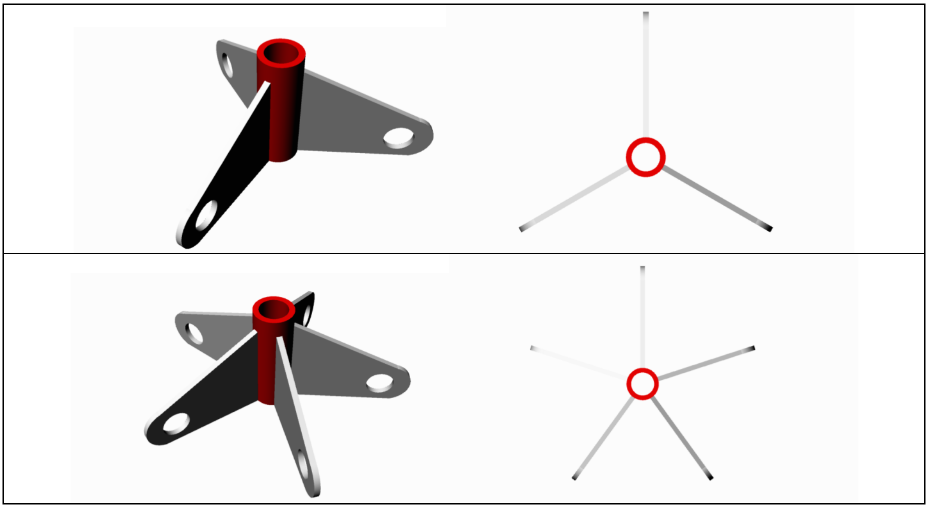


Figure 5.4: Joints with three and five fins for the dome structures

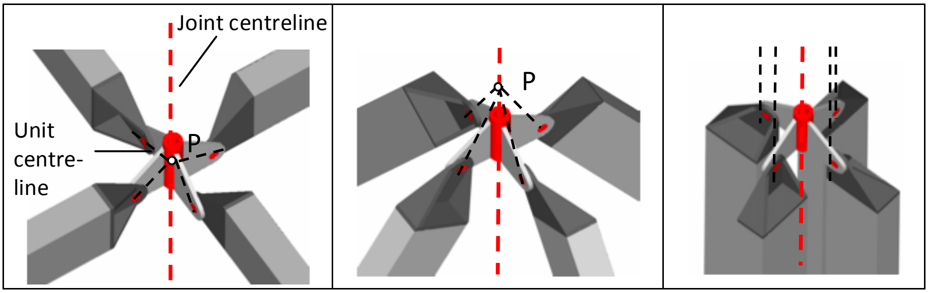


Figure 5.5: During deployment, the imaginary intersection point of the centrelines travels on the vertical centreline through the joint [De Temmerman, 2007]

5.2 Deployment

In case of the realisation of scissor-type deployable structures, one of the most important problems is concerned with the rational method of expanding. Not only proposed Escrig [1985] possibilities for joint elements, but he also presented three different solutions related to expanding mechanical devices: an electric motor by means of a screw (Figure 5.6a), a hydraulic system (Figure 5.6b) and a rope connecting distant joints by means of a tensor engine (Figure 5.6c).

Kokawa [1995, 1996] designed, analysed and constructed a zigzag-cable system. The cable passes through pulleys which are installed at the connection points between the scissor units. During winding up the cable by a winch, the structure expands and is forced to lift up. Figure 5.7 presents the cable arrangement and the basic idea of the changing form. According to computational simulations, it is shown numerically that the zigzag cable plays an important role in the point of structural efficiency improvement.

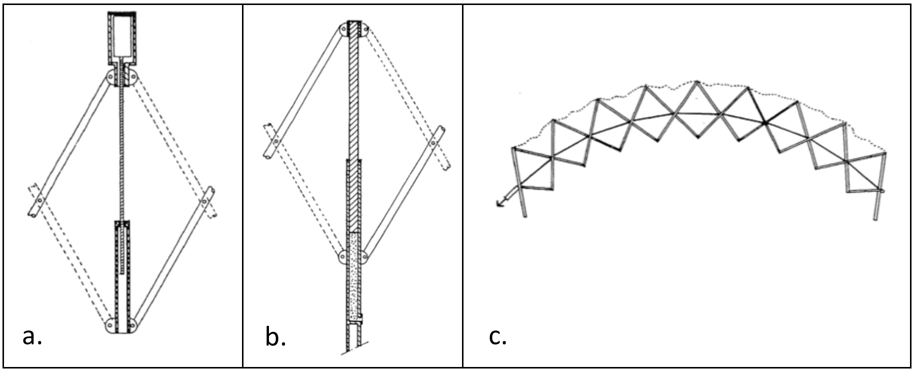


Figure 5.6: Expanding mechanical devices proposed by Escrig [1985]

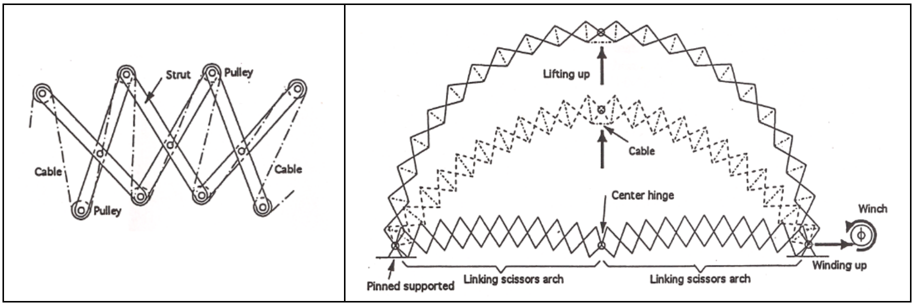


Figure 5.7: The zigzag-cable arrangement and the basic idea of deployment

After Kokawa [1995,1996], this research also examined the introduction of a simplified cable system in the barrel vault structure (cfr. Chapter 4). A zigzag-cable, called the active cable, runs through the mechanism, connecting upper and lower nodes along its path. Passive cables are upper cables connecting the upper nodes and lower cables connecting the lower nodes. As the active cable fulfils the deployment, the passive cables become tensioned, adding an extra stiffness to the structure and avoiding that the structure deploys too far (Figure 4.5). According to the structural results of case study 2 in previous chapter, it can be concluded that adding cables to the structure results in a lower total weight and introduces a self-stabilising phenomenon.

The hollow cylindrical hub of the proposed joint (red in Figure 5.3) can accept an active cable, guided by a pulley system, for stiffening the structure after deployment or for raising the membrane and bringing it under pre-tension[De Temmerman, 2007].

Instead of using steel cables it is common practice in the design and realisation of deployable structures to employ ropes. Flexibility makes ropes agile and easy to handle under all working conditions. They are not sensitive to corrosion and a water repellent protection keeps ropes from swelling when wet, and prevents them from becoming stiff and unmanageable. An additional important advantage compared to steel cables is a lower weight.

Table 5.1 compares the use of ropes instead of the steel active and passive cables in case study 2 as mentioned in section 4.5.3. The maximal normal force in the steel cables is 24kN (=2,4 tonnes) (Table 4.6). According to the specifications [Timko Ltd.©] a polypropylene 3 strand is chosen with a breaking strength of 3,5 tonnes.

The conclusion can be drawn that a significant reduction of cable weight of ***86%*** is acquired when using ropes instead of steel cables. This means that for case study 2 the weight of the structure will decrease from 29,3 kg/m^2 to 27,6 kg/m^2.

Table 5.1: Comparison between steel cables and ropes

Steel cables			
Maximal normal force [tonnes]			2,4
Cable	d [mm]	Length [m]	Weight [kg]
Active S235	8	170,2	67,2
Passive S235	12	626,6	556,3
Total		**796,8**	**623,5**
Ropes			
Breaking strength [tonnes]			3,5
Rope	d [mm]	Length [m]	Weight [kg]
Polypropylene 3	16	796,8	86,9
Reduction in weight [%]			**86**

5.3 Membrane

The membrane can be attached to the structure in several ways. After the supporting structure has been properly installed, the membrane can be fixed at an intermediate stage of the deployment process, to be deployed together with the scissors to the final configuration [Van Mele, 2008].
Or, the membrane can be raised toward the inner end nodes of the beam components when the structure is fully deployed, after which a basic level of pre-tension is introduced.
In the final configuration (Figure 5.8), after the supporting structure is secured, the membrane is correctly tensioned with ratchet elements.
Alternatively, instead of attaching the membrane to the inner nodes, the tensile cover could function as an outer layer, attached to the external nodes, providing weather protection for the structures as well [De Temmerman, 2007].

Creating a continuous roof surface requires a considerable amount of detailing, especially where the membrane and the scissors have to be connected. The design of these connections is even more complicated if the connection between the membrane and the scissors is temporary, like in transportable structures. Figure 5.9 presents a possibility to attach the membrane to a deployable structure with the illustration of a ratchet element. The structure presented in the latter figure is a realisation of the followed course "Active Form Constructions" of the Architectural Engineering Department at the VUB in the scope of the European project "Context-T".

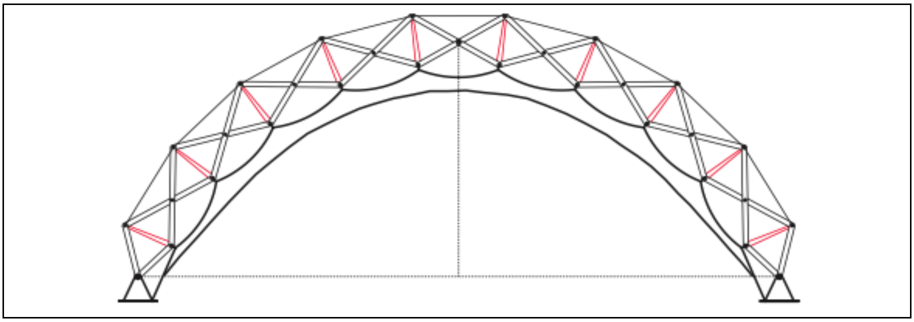


Figure 5.8: Scissor structure with an attached membrane connected at the lower corners [Van Mele, 2008]

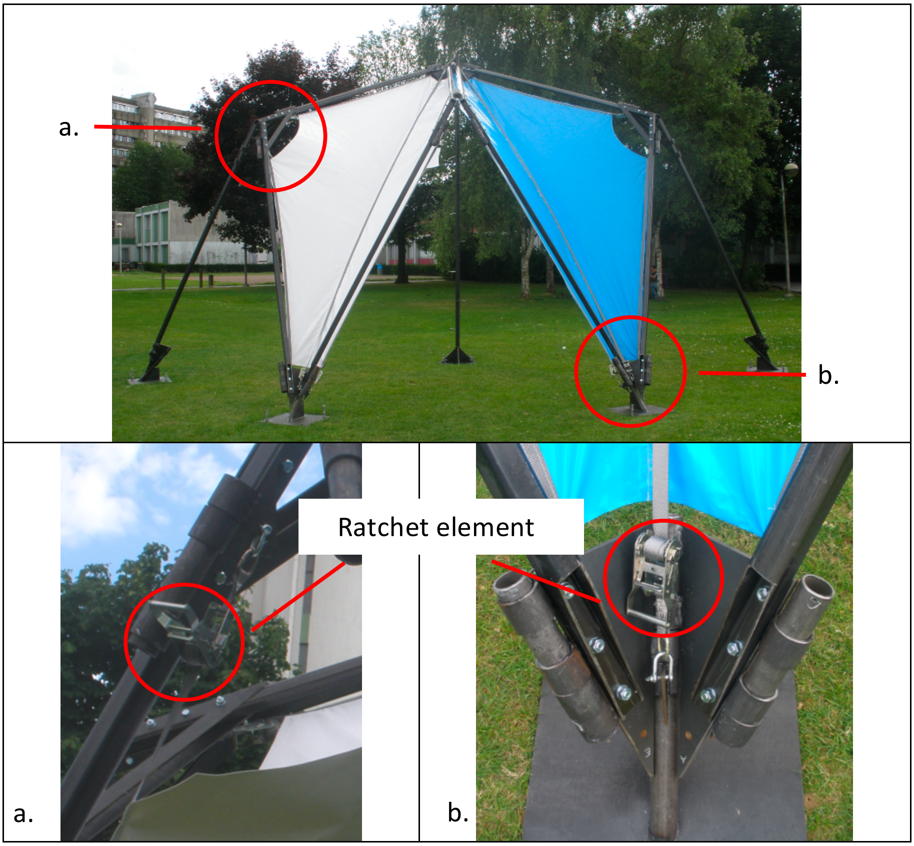


Figure 5.9: Deployable structure attached with a membrane and details of a ratchet element [VAC 2, 2009]

It is important to understand that there is a significant interaction between the membrane and the structure, especially when the membrane is fully stretched, in order to acquire the necessary pre-tension and the desired shape.

The pre-tensioned membrane provides an improvement of the overall stiffness resulting in a more self-stabilised structure (analogous to the introduction of cables). In practice the membrane can even function as a stabilising diaphragm, when it is stretched to the ground at the front or back as illustrated in Figure 5.10.

However, the scissor structure will be acquainted with an extra loading due to the forces required to pre-tension the membrane. This additional loading is especially significant at the edges of the membrane, where the membrane cables are situated.

Figure 5.10: Deployable structure in TaraTara Falcon [Grupo ESTRAN c.a., 2005]

6 Conclusions

The aim of the work presented in this dissertation was to develop a Universal Scissor Component (USC) for deployable structures and to propose different concepts leading to architecturally and structurally viable solutions for mobile applications. Furthermore, it was the objective to provide a methodology for designing and structurally analysing a new scissor component that results in generic deployable structures, well equipped to meet the demands of a rapidly changing society while embracing the concept of sustainable design.

By presenting a review of previous research on scissor structures, an insight has been given in the wide variety of possible shapes and configurations. Based on this knowledge, a new scissor concept could be explored. A preliminary feasibility study has been accomplished consisting of two main parts: (1) geometric design and (2) structural analysis. Successively, these two levels are evaluated and conclusions are drawn. Also, a number of suggestions for further work are provided.

6.1 Conclusions on the geometrical aspects

A preliminary analysis has shown that a triangular component has a larger structural efficiency compared to the standard scissor units - translational, polar and angulated – and has served as a first indication concerning the functionality and shape of the new USC.

A closer look has been taken on configurations of structures.
Firstly, cylindrical grids were chosen in this research, referred to as barrel vaults, for their simple and effective typology. They are obtained by rows of identical polar arches in the transverse direction, linked with translational

units in the longitudinal direction (paragraph 3.2.2.1). The scissor units obey the deployability constraint (section 2.3.1) resulting in a stress-free deployable barrel vault.
Secondly, deployable dome structures were considered, capable of retracting to their own perimeter.
Domes are not only architectural and structural viable configurations, they can also serve as a geometric transition to more exotic and interesting shapes thanks to the multi-functionality and advantages of angulated elements. The following polyhedra were considered for the dome structures: icosahedron, dodecahedron, icosidodecahedron, 'buckyball' and an adjusted rhombic triacontahedron. The deployable dome grids are obtained by substituting every edge of a polyhedron by angulated scissors satisfying the deployability condition.

A USC is designed with the ability to configure both barrel vaults and domes. To reach this possibility, it has been demonstrated that the new component is a combination of the three standard scissor units: translational, polar and angulated. Because hinge displacements have a dramatic influence on the structure shape, decisions were made concerning the different geometrical dimensions based on feasible hinge positions. This design process has resulted in a USC capable of configuring nineteen different architectural structures.

Further, the actual shape of the USC is determined. A decision is made to connect the hinges with beams resulting in a triangular truss-like component with three different beams: the strut, the mast and the oblique beams (paragraph 3.2.4).

Finally, the matter of deployment and kinematic implications is pursued, explaining the different deployment behaviour between barrel vaults and angulated domes. The hinged plate model, proposed by De Temmerman [2007], allowed gaining insight in the mobility of the mechanisms, which is necessary to understand to what degree constraints have to be added after deployment to turn them into load bearing structures. In case of the barrel vaults and the icosahedron dome, there is no need for additional constraints, other than the one needed to eliminate the rotational D.O.F. of the original scissor mechanism. In case of the other proposed polyhedra domes, additional constraints have to be considered due to the non-triangulation of the grid cells.

6.2 Conclusions on the structural aspects

In this part of the work an advance has been made by developing a methodology to investigate the feasibility of the designed concept. The largest proposed barrel vault, assuming to be representative and enabling to cover the remaining cases, is analysed structurally.

Relative small sections have been found (maximum 51mm), compared to the covered area, resulting in a feasible construction. The conclusion can be drawn that introducing cables not only helps developing a stable process of deployment, it also improves the structural performance and ultimately leads to a reduction in weight. The improvement of the structural performance of the system, due to the addition of active and passive cables, can be subject to discussion based on the numerical results. But it can be stated that there is no pre-tension applied in the cable segments, which on the contrary would be the case in the real application: by shortening more the active cable, a further deployment is created with extra pre-tension in the passive cables, contributing to an increase of the global stiffness of the structure.

In addition, it has been described that in the investigated case study a further reduction of cable weight of 86% can be acquired when using ropes instead of steel cables.

In previous researches, it is stated that traditional scissor elements show a low to medium structural efficiency due to existing bending moments. In this section it has been proven that the designed USC rather excludes bending moments, increasing the structural efficiency of the structure.

In Chapter 3, concerned with the geometric design, an important decision has been made: a length of 2m is chosen for the USC because of the resulting range from low to high spans for multifunctional deployable structures. This choice can be justified by the structural results in Chapter 4. In the two case studies the weight of a single Universal Scissor Component remains far below 20 kg. The weight values are acceptable for manual handling of charges by workers. A length more than 2m could involve manual lifting problems.

A simplified method has been applied due to a missing appropriate software tool in which all components can be implemented and analysed. A preliminary analysis was sufficient for the projected results within the scope of this research. SCIA ESA-PT was readily available software, generally aimed at standard constructions. However, it was necessary to make model assumptions to examine this type of non-standard structure. A multitude of simulation problems were encountered in the course of this research. Solving them was time-consuming, but ultimately the research has led to acceptable results.

It can be expected that nearly all of the USC beams are over-dimensioned, because the optimisation process focused on the maximal loaded beam in the whole structure. In addition, the structural analysis is done on the largest barrel vault. The sections resulting from the structural design are again over-dimensioned in case of the alternative smaller geometrical configurations (cfr. section 3.2.6). And finally it can be concluded that even the total weight is significant (although acceptable) compared to other mobile constructions. But the main goal of this thesis has been to design and analyse the feasibility of a new type of scissor component, which is identical in multiple architectural structures, creating a generic design solution.

6.3 Reflections and further work

A simplified approach, used for the structural analysis, has been sufficient within the framework of a preliminary design to evaluate the feasibility of the concepts. Although acceptable results are encountered, more details should be explored to justify the overall feasibility. Is a barrel vault, enclosing 300 m^2, with a weight of more than 9000 kg, required or even manageable in the context of mobile architectural applications?

A profound analysis is needed in order to investigate the main question, which is rather “Should it be built?” than “Can it be built?”. Therefore, it is preferable to use an integrated model which takes the mutual response of the membrane surface and the beam structure into account. Also the attachment and tensioning of the membrane is a complex process, implying structural and feasibility implications. Since the joints, which have been excluded from the analysis, constitute a major part of the design, a detailed study is needed.

The element length of the USC is currently 2m. It could be investigated what an optimal length could be, both from a structural point of view, but also keeping in mind the architectural and practical implications.

Similar to [De Temmerman, 2007] it would be beneficial to examine the impact of adding cross-cables in each grid cell. This leads to a significant increase in stiffness, resulting in a decrease in section dimensions and thus weight.

Furthermore, it could be interesting to see to what extent the proposed concept can be broadened to multi-angulated elements (see paragraph 2.2.3) in terms of geometry, technology and structural performance.

A thorough understanding of the cable system could also lead to an efficient improvement of the structure. It could be explored to what extent the further deployment by means of the active cable system, thus by introducing pre-tension in the passive cable system, can contribute to the overall structural performance and possible weight optimisation.
But due to the possibility that this system could lead to complicated joints and an unmanageable structure, this exploration should be combined inevitably with a study focussing on the complexity when introducing a cable system.
Generally, a thorough consideration of the pro's and contra's could favour the feasibility of the project.

And also, future work can be done in terms of 'sustainability'. This research about a universal component in the field of deployable structures, creating generic solutions could be widened and further designed according to a fully and detailed '4D-concept'.

Deployable structures have become increasingly popular, but few have been realised successfully. Further work can consist of a better understanding of the design parameters and the related allowable tolerances and imperfections, in order to respect the architectural function of the structure and to guarantee a successful deployment and folding. Their further development calls for research into fundamental issues regarding the shape of the constitutive elements, the connections, the membrane, the deployment behaviour, and the structural performance, both during and after the transformation process.
This would allow spreading such structures worldwide, implying a green and sustainable impact on the environment.

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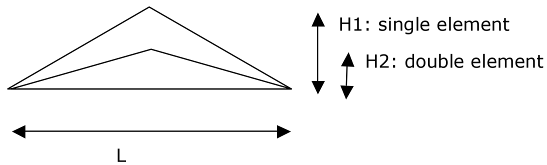
Appendix 1: Domes

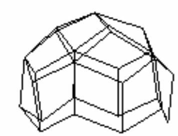
DOMES - for angulated element



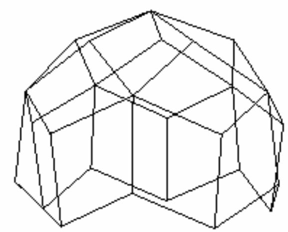
1. Adjusted Rhombic Triacontahedron

L (m)	H1(cm)	H2(cm)	S(m) single dome	H(m) single dome	S(m) double dome	H(m) double dome	S(m) double dome low	H(m) double dome low
1,0	16,9	8,2	3,1	2,3	6,1	4,5	5,8	2,5
1,2	20,3	9,8	3,7	2,7	7,3	5,4	6,9	3,0
1,5	25,4	12,3	4,6	3,4	9,2	6,7	8,6	3,5
1,8	30,4	14,8	5,5	4,1	11,0	8,0	10,4	4,5
2,0	33,8	16,4	6,1	4,5	12,2	8,9	11,5	5,0

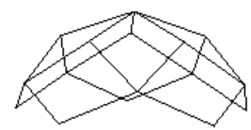
Dome with single element



Dome with double element

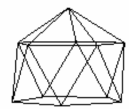
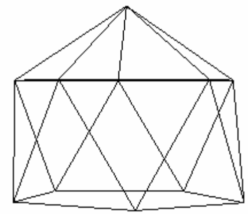
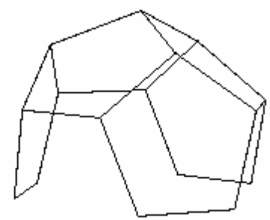
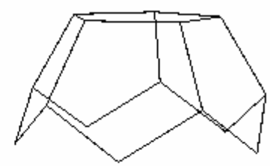


Lower Dome with double element



2. Icosahedron

L (m)	H1(cm)	H2(cm)	S(m) single dome	H(m) single dome	S(m) double dome	H(m) double dome
1,0	30,9	14,2	1,6	1,4	3,1	2,7
1,2	37,1	17,0	1,9	1,7	3,7	3,2
1,5	46,4	21,3	2,4	2,1	4,7	4,0
1,8	55,6	25,6	2,9	2,5	5,6	4,8
2,0	61,8	28,4	3,2	2,8	6,2	5,3

Dome with single element	Dome with double element	Lower Dome with double element
		
		

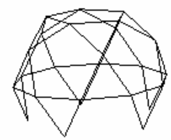
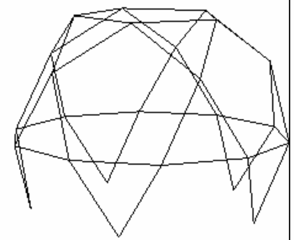
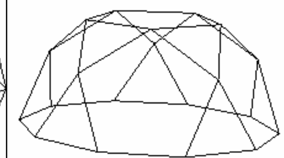
3. Dodecahedron

L (m)	H1(cm)	H2(cm)	S(m) single dome	H(m) single dome	S(m) double dome	H(m) double dome	S(m) double dome low	H(m) double dome low
1,0	19,1	9,2	2,6	1,9	5,2	3,7	5,2	2,7
1,2	22,9	11,0	3,1	2,3	6,2	4,4	6,2	3,2
1,5	28,7	13,8	3,9	2,8	7,7	5,6	7,7	4,1
1,8	34,4	16,6	4,7	3,4	9,3	6,7	9,3	4,9
2,0	38,2	18,4	5,2	3,8	10,3	7,4	10,3	5,4

4. Icosidodecahedron

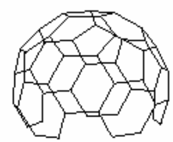
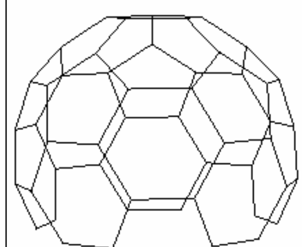
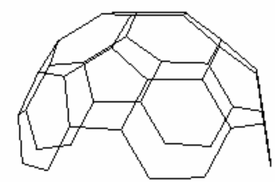
L (m)	H1(cm)	H2(cm)	S(m) single dome	H(m) single dome	S(m) double dome	H(m) double dome	S(m) double dome low	H(m) double dome low
1,0	16,2	7,9	2,6	2,2	5,2	4,4	6,4	2,7
1,2	19,4	9,5	3,1	2,7	6,2	5,3	7,7	3,3
1,5	24,3	11,9	3,9	3,3	7,8	6,6	9,6	4,1
1,8	29,2	14,2	4,7	4,0	9,3	7,9	11,5	4,9
2,0	32,4	15,8	5,2	4,5	10,3	8,8	12,8	5,4

Dome with single element | Dome with double element | Lower Dome with double element

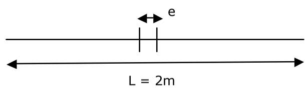
5. Bucky Ball

L (m)	H1(cm)	H2(cm)	S(m) single dome	H(m) single dome	S(m) double dome	H(m) double dome	S(m) double dome low	H(m) double dome low
1,0	10,3	5,1	4,2	3,4	8,5	6,8	9,2	5,5
1,2	12,4	6,1	5,1	4,1	10,1	8,2	11,1	6,6
1,5	15,5	7,7	6,4	5,1	12,7	10,3	13,8	8,3
1,8	18,5	9,2	7,6	6,2	15,2	12,3	16,6	9,9
2,0	20,6	10,2	8,5	6,8	16,9	13,7	18,4	11,0

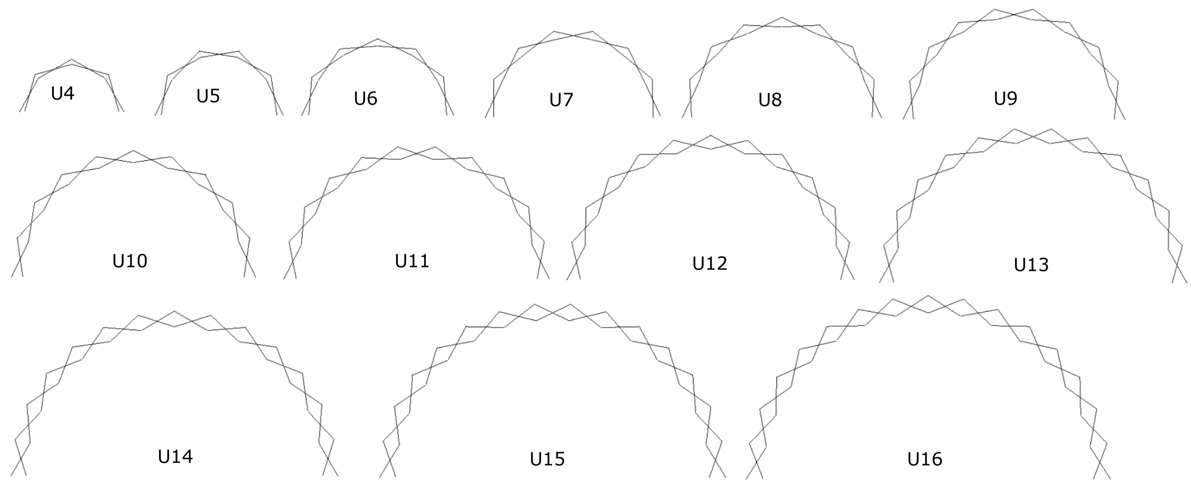
Appendix 2: Barrel Vaults

ARCHES - for polar element



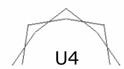
1. Excentricity: e=5cm

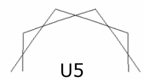
U	S(m)	T(m)	
4	4,9292	0,2594	**Too far deployed**
5	6,077	0,3198	**Too far deployed**
6	7,2165	0,3798	**Too far deployed**
7	8,3407	0,4399	
8	9,4452	0,4971	
9	10,5266	0,554	
10	11,5822	0,6096	
11	12,6099	0,6639	
12	13,6081	0,7162	
13	14,5754	0,7671	
14	15,511	0,8164	
15	16,4142	0,8639	
16	17,2846	0,9097	



2. Excentricity: e=10cm

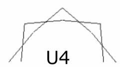
U	S(m)	T(m)	
4	4,5723	0,508	**Too far deployed**
5	5,5672	0,6186	
6	6,5157	0,724	





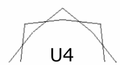
3. Excentricity: e=12cm

U	S(m)	T(m)	
4	4,4175	0,6024	**Too far deployed**



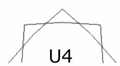
4. Excentricity: e=13cm

U	S(m)	T(m)	
4	4,3382	0,6482	**Too far deployed**



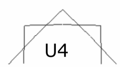
5. Excentricity: e=14cm

U	S(m)	T(m)	
4	4,2579	0,6932	**Too far deployed**



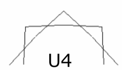
6. Excentricity: e=15cm

U	S(m)	T(m)	
4	4,1769	0,7371	**Too far deployed**



7. Excentricity: e=16cm

U	S(m)	T(m)
4	4,0951	0,7800



Appendix 3: Calculation Notes Case Study 1

1. Steel control: Mast

Staalcontrole

Niet-lineaire berekening,Extreem : Globaal
Selectie : Alle
Niet-lineaire combinaties : NLULS4
Doorsnede : CS4 - MSRR51.0x2.9

EC3 : EN 1993 Code Check

Staaf S2844	MSRR51.0x2.9	S 235	NLULS4	0.77

NEd [kN]	Vy,Ed [kN]	Vz,Ed [kN]	TEd [kNm]	My,Ed [kNm]	Mz,Ed [kNm]
Studentenversie	*Studentenversie*	*Studentenversie*	*Studentenversie*	*Studentenversie*	*Studenten
-12.63	1.08	0.27	-0.09	-0.00	0.16

Kritische controle op positie 2.00 m

Knikparameters	yy	zz	
Studentenversie *Studentenversie*	*Studentenversie*	*Studentenversie*	*Studentenversie* *S
type	geschoord	geschoord	
Slankheid	49.35	33.98	
Gereduceerdeslankheid	0.53	0.36	
Knikkromme	a	a	
Imperfectie	0.21	0.21	
Knikfactor (omega_buc)	0.92	0.96	
Lengte	1.00	0.95	m
lef/lsys	0.84	0.61	
Kniklengte	0.84	0.58	m
Kritische Euler belastingen	372.75	786.14	kN

LTB		
Studentenversie	*Studentenversi	
Kiplengte	0.95	m
k	1.00	
kw	1.00	
C1	1.86	
C2	0.01	
C3	0.94	

SPANNINGSCONTROLE	
Studentenversie *Studentenversie* *Studentenversie* *Studentenversie	
Controle druk	0.12 < 1
Torsiecontrole	0.06 < 1
Controle afschuiving (Vy)	0.03 < 1
Controle afschuiving (Vz)	0.01 < 1
Controle buigend moment (My)	0.00 < 1
Controle buigend moment (Mz)	0.10 < 1
M	0.10 < 1

STABILITEITSCONTROLE	
Studentenversie *Studentenversie* *Studentenversie* *Student	
Knik	0.13 < 1
KIP	0.00 < 1
Druk + moment	0.66 < 1
Druk + moment	0.77 < 1

2. Steel control: Oblique beam

Staalcontrole

Niet-lineaire berekening,Extreem : Globaal
Selectie : Alle
Niet-lineaire combinaties : NLULS1
Doorsnede : CS2 - MSRR51.0x3.2

EC3 : EN 1993 Code Check

Staaf S307	MSRR51.0x3.2	S 235	NLULS1	0.97

NEd [kN]	Vy,Ed [kN]	Vz,Ed [kN]	TEd [kNm]	My,Ed [kNm]	Mz,Ed [kNm]
Studentenversie	*Studentenversie*	*Studentenversie*	*Studentenversie*	*Studentenversie*	*Studenten
-24.30	-31.45	-39.81	-0.03	0.21	0.25

Kritische controle op positie 1.07 m

Knikparameters	yy	zz	
Studentenversie *Studentenversie*	*Studentenversie*	*Studentenversie*	*S
type	geschoord	geschoord	
Slankheid	63.20	45.69	
Gereduceerdeslankheid	0.67	0.49	
Knikkromme	a	a	
Imperfectie	0.21	0.21	
Knikfactor (omega_buc)	0.86	0.93	
Lengte	1.07	1.07	m
lef/lsys	1.00	0.72	
Kniklengte	1.07	0.77	m
Kritische Euler belastingen	249.61	477.63	kN

LTB		
Studentenversie	*Studentenversi	
Kiplengte	1.07	m
k	1.00	
kw	1.00	
C1	2.15	
C2	0.00	
C3	0.85	

SPANNINGSCONTROLE	
Studentenversie *Studentenversie* *Studentenversie* *Studentenversie	
Controle druk	0.22 < 1
Torsiecontrole	0.02 < 1
Controle afschuiving (Vy)	0.76 < 1
Controle afschuiving (Vz)	0.97 < 1
Controle buigend moment (My)	0.12 < 1
Controle buigend moment (Mz)	0.15 < 1
M	0.60 < 1

STABILITEITSCONTROLE	
Studentenversie *Studentenversie* *Studentenversie* *Studente	
Knik	0.25 < 1
KIP	0.12 < 1
Druk + moment	0.43 < 1
Druk + moment	0.43 < 1

3. Steel control: Strut

Staalcontrole

Niet-lineaire berekening,Extreem : Globaal
Selectie : Alle
Niet-lineaire combinaties : NLULS4
Doorsnede : CS3 - MSRR21.3x2.3

EC3 : EN 1993 Code Check

Staaf S3577	MSRR21.3x2.3	S 235	NLULS4	0.73

NEd [kN]	Vy,Ed [kN]	Vz,Ed [kN]	TEd [kNm]	My,Ed [kNm]	Mz,Ed [kNm]
Studentenversie	*Studentenversie*	*Studentenversie*	*Studentenversie*	*Studentenversie*	*Studenten
-8.46	0.37	-0.00	-0.00	-0.00	0.10

Kritische controle op positie 0.38 m

Knikparameters	yy	zz	
Studentenversie *Studentenversie*	*Studentenversie*	*Studentenversie*	*St
type	geschoord	geschoord	
Slankheid	56.38	38.98	
Gereduceerdeslankheid	0.60	0.42	
Knikkromme	a	a	
Imperfectie	0.21	0.21	
Knikfactor (omega_buc)	0.89	0.95	
Lengte	0.38	0.38	m
lef/lsys	1.00	0.69	
Kniklengte	0.38	0.26	m
Kritische Euler belastingen	89.34	186.88	kN

LTB		
Studentenversie	*Studentenversi	
Kiplengte	0.38	m
k	1.00	
kw	1.00	
C1	1.15	
C2	0.60	
C3	0.53	

SPANNINGSCONTROLE	
Controle druk	0.26 < 1
Controle afschuiving (Vy)	0.03 < 1
Controle afschuiving (Vz)	0.00 < 1
Controle buigend moment (My)	0.00 < 1
Controle buigend moment (Mz)	0.50 < 1
M	0.54 < 1

STABILITEITSCONTROLE	
Knik	0.30 < 1
KIP	0.00 < 1
Druk + moment	0.56 < 1
Druk + moment	0.73 < 1

4. Displacements

Vervormingen van staaf

Niet-lineaire berekening, Extreem : Globaal
Klasse : All NLSLS

BG	Staaf	dx [m]	ux [mm]	uy [mm]	uz [mm]	fix [mrad]	fiy [mrad]	fiz [mrad]
NLSLS5	S1782	0,000	**-33,8**	-0,1	3,9	0,2	-0,4	0,1
NLSLS2	S1884	0,000	**34,0**	0,2	-3,9	-0,3	0,0	-0,1
NLSLS2	S1734	0,000	-3,3	**-30,2**	17,5	0,9	0,1	0,8
NLSLS2	S2343	1,070	3,4	**30,2**	-16,7	-0,1	-0,4	0,9
NLSLS2	S1920	2,000	-4,5	0,2	**-34,7**	0,3	0,1	0,3
NLSLS5	S1845	1,070	-1,1	0,5	**34,0**	-0,4	-2,2	-0,5
NLSLS4	S148	2,000	-6,0	0,7	10,0	**-7,9**	-6,5	-3,1
NLSLS4	S929	0,000	5,7	11,9	1,6	**9,0**	-4,5	0,0
NLSLS4	S3337	2,000	0,0	0,0	0,0	-3,9	**-9,9**	2,0
NLSLS2	S3199	2,000	0,0	0,0	0,0	1,1	**8,9**	-0,4
NLSLS4	S152	0,096	-3,4	2,6	-7,8	-2,3	-6,7	**-6,4**
NLSLS4	S3577	0,382	-7,8	-2,2	-0,1	-0,3	-7,2	**6,6**

5. Reactions

Reacties

Niet-lineaire berekening,Extreem : Globaal
Selectie : Alle
Klasse : All NLULS

Steunpunt	BG	Rx [kN]	Ry [kN]	Rz [kN]	Mx [kNm]	My [kNm]	Mz [kNm]
Sn20/K3077	NLULS4	**-13,37**	-9,32	45,92	0,00	0,00	0,00
Sn2/K375	NLULS6	**13,09**	10,37	43,61	0,00	0,00	0,00
Sn22/K3438	NLULS4	-7,53	**-32,07**	58,00	0,00	0,00	0,00
Sn1/K278	NLULS1	7,82	**31,93**	57,26	**0,00**	**0,00**	**0,00**
Sn1/K278	NLULS2	-7,07	-19,36	**-34,99**	0,00	0,00	0,00
Sn3/K183	NLULS4	-8,92	29,08	**62,76**	0,00	0,00	0,00

6. Stability: Critical loading coefficient

Kritische belastingcoëfficiënten

Kritische belastingcoëfficiënten

N -	f []
Stabiliteitcombinatie : S1	
1	5,06
2	5,15
3	5,32
4	5,89
Stabiliteitcombinatie : S4	
1	5,11
2	5,47
3	5,57
4	5,73

Appendix 4: Calculation Notes Case Study 2

1. Steel control: Mast

Staalcontrole

Niet-lineaire berekening,Extreem : Globaal
Selectie : Alle
Niet-lineaire combinaties : NLULS2
Doorsnede : CS4 - MSRR42.4x2.6

EC3 : EN 1993 Code Check

Staaf S1051	MSRR42.4x2.6	S 235	NLULS2	0.85

NEd [kN]	Vy,Ed [kN]	Vz,Ed [kN]	TEd [kNm]	My,Ed [kNm]	Mz,Ed [kNm]
Studentenversie	*Studentenversie*	*Studentenversie*	*Studentenversie*	*Studentenversie*	*Studenten
-20.28	-0.70	-0.17	-0.10	0.00	0.29

Kritische controle op positie 0.00 m

Knikparameters	yy	zz	
Studentenversie *Studentenversie* *Studenten	versie* *Studentenversie*	*Studentenversie* *S	
type	geschoord	geschoord	
Slankheid	66.56	46.50	
Gereduceerdeslankheid	0.71	0.50	
Knikkromme	a	a	
Imperfectie	0.21	0.21	
Knikfactor (omega_buc)	0.84	0.93	
Lengte	1.00	1.05	m
lef/lsys	0.94	0.62	
Kniklengte	0.94	0.66	m
Kritische Euler belastingen	152.06	311.51	kN

LTB		
Studentenversie	*Studentenversi	
Kiplengte	1.05	m
k	1.00	
kw	1.00	
C1	1.88	
C2	0.00	
C3	0.94	

SPANNINGSCONTROLE	
Controle druk	0.27 < 1
Torsiecontrole	0.11 < 1
Controle afschuiving (Vy)	0.03 < 1
Controle afschuiving (Vz)	0.01 < 1
Controle buigend moment (My)	0.00 < 1
Controle buigend moment (Mz)	0.30 < 1
M	0.32 < 1

STABILITEITSCONTROLE	
Knik	0.31 < 1
KIP	0.00 < 1
Druk + moment	0.75 < 1
Druk + moment	0.85 < 1

2. Steel control: Oblique beam

Staalcontrole

Niet-lineaire berekening,Extreem : Globaal
Selectie : Alle
Niet-lineaire combinaties : NLULS4
Doorsnede : CS2 - MSRR33.7x4.5

EC3 : EN 1993 Code Check

Staaf S3216	MSRR33.7x4.5	S 235	NLULS4	0.91

NEd [kN]	Vy,Ed [kN]	Vz,Ed [kN]	TEd [kNm]	My,Ed [kNm]	Mz,Ed [kNm]
Studentenversie	*Studentenversie*	*Studentenversie*	*Studentenversie*	*Studentenversie*	*Studenten
-20.56	-10.92	-30.78	-0.08	0.15	-0.01

Kritische controle op positie 1.07 m

Knikparameters	yy	zz	
Studentenversie *Studentenversie* *Studenten	versie* *Studenten	versie* *Studentenversie*	*St
type	geschoord	geschoord	
Slankheid	102.55	73.20	
Gereduceerdeslankheid	1.09	0.78	
Knikkromme	a	a	
Imperfectie	0.21	0.21	
Knikfactor (omega_buc)	0.60	0.81	
Lengte	1.07	1.07	m
lef/lsys	1.00	0.71	
Kniklengte	1.07	0.76	m
Kritische Euler belastingen	81.39	159.76	kN

LTB		
Studentenversie	*Studentenversi	
Kiplengte	1.07	m
k	1.00	
kw	1.00	
C1	1.00	
C2	0.00	
C3	1.00	

SPANNINGSCONTROLE	
Studentenversie *Studentenversie* *Studentenversie* *Studentenversi	
Controle druk	0.21 < 1
Torsiecontrole	0.10 < 1
Controle afschuiving (Vy)	0.32 < 1
Controle afschuiving (Vz)	0.91 < 1
Controle buigend moment (My)	0.16 < 1
Controle buigend moment (Mz)	0.01 < 1
M	0.12 < 1

STABILITEITSCONTROLE	
Studentenversie *Studentenversie* *Studentenversie* *Student	
Knik	0.35 < 1
KIP	0.16 < 1
Druk + moment	0.62 < 1
Druk + moment	0.49 < 1

3. Steel control: Strut

Staalcontrole

Niet-lineaire berekening,Extreem : Globaal
Selectie : Alle
Niet-lineaire combinaties : NLULS4
Doorsnede : CS3 - MSRR21.3x2.3

EC3 : EN 1993 Code Check

Staaf S790	MSRR21.3x2.3	S 235	NLULS4	0.50

NEd [kN]	Vy,Ed [kN]	Vz,Ed [kN]	TEd [kNm]	My,Ed [kNm]	Mz,Ed [kNm]
Studentenversie	*Studentenversie*	*Studentenversie*	*Studentenversie*	*Studentenversie*	*Studenten
-5.02	0.27	0.00	-0.00	-0.00	-0.03

Kritische controle op positie 0.00 m

Knikparameters	yy	zz	
Studentenversie *Studentenversie* *Studenten	versie* *Studentenversie*	*Studentenversie*	*S
type	geschoord	geschoord	
Slankheid	56.38	45.70	
Gereduceerdeslankheid	0.60	0.49	
Knikkromme	a	a	
Imperfectie	0.21	0.21	
Knikfactor (omega_buc)	0.89	0.93	
Lengte	0.38	0.38	m
lef/lsys	1.00	0.81	
Kniklengte	0.38	0.31	m
Kritische Euler belastingen	89.34	135.93	kN

LTB		
Studentenversie	*Studentenversi	
Kiplengte	0.38	m
k	1.00	
kw	1.00	
C1	1.14	
C2	0.51	
C3	0.53	

SPANNINGSCONTROLE	
Controle druk	0.16 < 1
Torsiecontrole	0.02 < 1
Controle afschuiving (Vy)	0.02 < 1
Controle afschuiving (Vz)	0.00 < 1
Controle buigend moment (My)	0.00 < 1
Controle buigend moment (Mz)	0.16 < 1
M	0.16 < 1

STABILITEITSCONTROLE	
Knik	0.18 < 1
KIP	0.00 < 1
Druk + moment	0.37 < 1
Druk + moment	0.50 < 1

4. Steel control: Passive cable

Staalcontrole

Niet-lineaire berekening,Extreem : Globaal
Selectie : Alle
Niet-lineaire combinaties : NLULS4
Doorsnede : CS5 - RD12

EC3 : EN 1993 Code Check

Staaf S3883	RD12	S 235	NLULS4	0.90

NEd [kN]	Vy,Ed [kN]	Vz,Ed [kN]	TEd [kNm]	My,Ed [kNm]	Mz,Ed [kNm]
Studentenversie	*Studentenversie*	*Studentenversie*	*Studentenversie*	*Studentenversie*	*Studenten
23.84	-0.00	-0.00	-0.00	-0.00	-0.00

Kritische controle op positie 1.87 m

LTB		
Studentenversie	*Studentenversi	
Kiplengte	1.87	m
k	1.00	
kw	1.00	
C1	1.00	
C2	0.00	
C3	1.00	

last in zwaartepunt

SPANNINGSCONTROLE	
Studentenversie *Studentenversie* *Studentenversie*	*Stud
Controle normaalkracht	0.90 < 1
M	0.90 < 1

5. Steel control: Active cable

Staalcontrole

Niet-lineaire berekening,Extreem : Globaal
Selectie : Alle
Niet-lineaire combinaties : NLULS2
Doorsnede : CS6 - RD8

EC3 : EN 1993 Code Check

Staaf S4323	RD8	S 235	NLULS2	0.89

NEd [kN]	Vy,Ed [kN]	Vz,Ed [kN]	TEd [kNm]	My,Ed [kNm]	Mz,Ed [kNm]
Studentenversie	*Studentenversie*	*Studentenversie*	*Studentenversie*	*Studenten	
10.48	-0.00	-0.00	-0.00	-0.00	-0.00

Kritische controle op positie 0.92 m

LTB		
Studentenversie	*Studentenversi	
Kiplengte	0.92	m
k	1.00	
kw	1.00	
C1	1.00	
C2	0.00	
C3	1.00	

last in zwaartepunt

SPANNINGSCONTROLE	
Studentenversie *Studentenversie* *Studentenversie*	*Stud
Controle normaalkracht	0.89 < 1
M	0.89 < 1

6. Displacements

Vervormingen van staaf

Niet-lineaire berekening,Extreem : Globaal
Klasse : All NLSLS

BG	Staaf	dx [m]	ux [mm]	uy [mm]	uz [mm]	fix [mrad]	fiy [mrad]	fiz [mrad]
NLSLS2	S1785	0,000	**-24,8**	0,2	0,9	-0,6	-0,6	-0,2
NLSLS2	S1088	0,382	**27,1**	-0,3	-0,6	4,1	-0,1	-3,2
NLSLS2	S1028	0,000	-2,2	**-25,1**	10,1	1,7	-2,0	0,5
NLSLS2	S1629	1,070	3,3	**24,9**	-9,1	0,1	-0,5	0,7
NLSLS2	S1920	2,000	-4,4	0,1	**-27,8**	0,6	0,6	0,3
NLSLS2	S1798	0,000	10,2	0,5	**25,0**	1,9	0,3	0,3
NLSLS1	S305	2,000	0,0	0,0	0,0	**-10,1**	-3,7	4,6
NLSLS4	S3584	0,000	-4,4	-11,2	0,4	**8,2**	-4,3	5,9
NLSLS4	S3585	1,070	0,0	0,0	0,0	-6,9	**-9,6**	6,8
NLSLS2	S307	1,070	0,0	0,0	0,0	3,5	**8,5**	-3,6
NLSLS2	S393	0,000	3,4	1,2	21,1	4,4	5,9	**-7,5**
NLSLS4	S3581	0,125	-4,3	-10,0	-0,6	4,8	-4,7	**9,7**

7. Reactions

Reacties

Niet-lineaire berekening,Extreem : Globaal
Selectie : Alle
Klasse : All NLULS

Steunpunt	BG	Rx [kN]	Ry [kN]	Rz [kN]	Mx [kNm]	My [kNm]	Mz [kNm]
Sn20/K3077	NLULS4	**-13,86**	-9,92	43,57	0,00	0,00	0,00
Sn2/K375	NLULS6	**13,03**	8,33	38,06	0,00	0,00	0,00
Sn22/K3438	NLULS1	-7,96	**-24,96**	40,20	0,00	0,00	0,00
Sn3/K183	NLULS1	-8,30	**22,29**	41,76	0,00	0,00	0,00
Sn1/K278	NLULS2	-6,78	-13,30	**-27,28**	**0,00**	**0,00**	**0,00**
Sn20/K3077	NLULS1	-13,67	-10,48	**45,11**	0,00	0,00	0,00

8. Stability: Critical loading coefficient

Kritische belastingcoëfficiënten

Kritische belastingcoëfficiënten

N -	f []
Stabiliteitcombinatie : S4	
1	5,93
2	6,22
3	6,46
4	6,46
Stabiliteitcombinatie : S2	
1	-5,62
2	-5,50
3	-5,32
4	-5,03