Quantifying the flexibility of residential electricity demand in 2050: a bottom-up approach

Jonas Engels, *Student, KU Leuven,* Dries Guldentops, *Student, KU Leuven*

***Abstract*—This paper presents a method to quantify the flexibility of automatic demand response applied to residential electricity demand using price elasticities. First, a stochastic bottom-up model of flexible electricity demand in 2050 is presented. Three types of flexible devices are implemented: electrical heating, EVs and wet appliances. They are allocated into houses which are grouped together into neighbourhoods. Each house schedules its flexible demand w.r.t. a varying price signal, in order to minimize electricity cost. Thus, for each price signal, each neighbourhood yields a corresponding electricity consumption pattern. Through a regression analysis, a relationship between a change in electricity price and electricity consumption, relative to a reference scenario, can be obtained. This allows determining the price elasticities. A selective regression is applied which enables to determine the time interval in which price changes have an influence on the demand. By performing a Monte Carlo simulation, the characteristics of an average neighbourhood can be determined. This allows to scale up the results to the level of a country. The simulation is performed for the four different seasons and both weekdays and weekends.**

The results show that the electric energy demand will double and peak power demand could increase by a factor 5 to 8 compared to the one observed today. The elasticity matrices point out that most flexibility is available in winter and least in summer. The maximum amount of time that demand is shifted is found to be 7 hours, given the implemented strategy.

***Index Terms*—Smart grids, demand response, RTP, flexibility, price elasticities**

I. Introduction

In order to limit the global temperature rise to an average of 2°C, the leaders of the European Union and the G8 announced to reduce greenhouse gas emissions at least 80 % below 1990 levels by 2050 [1]. The European Commission translated this objective into two separate roadmaps: the Energy Roadmap 2050 [2] and the roadmap for moving to a low-carbon economy in 2050 [3], both implying near carbon neutrality for the power sector in 2050. Several major stakeholders in the energy sector have issued studies, either predating or following the Commissions roadmaps, with the goal of establishing scenarios for reaching this low-carbon energy system by 2050 [1], [4], [5]. In the outcome of these studies, a significant share of the electricity production in 2050 comes from renewables, ranging between 40 % and 100 %. To cope with these intermittent sources, the power system is expected to need more operational flexibility to balance demand and supply. With the advent of smart grids, an active participation of the demand side through the use of Demand Response (DR) programs is a promising source of flexibility. Several DR programs exist, a good overview is given in [6]. Of these programs, many economists believe that real time pricing (RTP) is the most direct and efficient DR mechanism and therefore should be the focus of policy makers [7]. Different modelling approaches exist to represent 'flexibility'. However, they all suffer from some drawbacks. Some approaches represent the flexibility very simplified as a given percentage of the energy demand that can be shifted [1], [8], [9]. Others lose valuable temporal information [10] or only consider one type of flexible device [11]–[14]. Still others are more economically oriented, representing flexibility by price elasticities. They do not take into account the technical characteristics of different flexible devices [15], [16]. A major challenge for the system operator is to integrate this flexibility ex-ante in the market [17]. For this purpose, however, the available flexibility must first be quantified.

This paper presents a new model to quantify the flexibility of automatic demand response of residential electricity demand using price elasticities. The model is built up using a bottom-up approach, meaning that the technical characteristics of the flexible devices are taken into consideration. A fully carbon-neutral scenario is assumed with a complete electrification. All residential customers are assumed to participate in a RTP program. In this paper, this has been carried out for Belgian residential electricity demand.

The remainder of this paper is organized as follows. First, section II discusses the models of the flexible devices taken into account and explains the cost-minimization problem. section III describes the applied regressions in order to come to the elasticity matrices. To find the average values of these elasticity matrices, a Monte Carlo simulation is performed and explained in section IV. The results of this Monte Carlo simulation are presented and discussed in section V, followed by a conclusion in section VI.

II. Demand Model

The residential electricity demand of a household can be divided in two different types: non-flexible demand and flexible demand, which can be shifted in time. Non-flexible demand and occupancy profiles are modelled based upon [18] and scaled to 2050 according to [9]. Three types of flexible devices are considered in this paper. The data and models of these devices will be briefly explained in the subsections below.

A. Flexible Devices

Flexible devices can be divided in 'discrete devices' and 'continuous devices'. Discrete devices must run a complete predefined cycle once started and cannot be interrupted. This paper will consider *wet appliances*. Continuous devices are not restricted in their load cycle. They can be interrupted as much as desired and can draw a varying power. They will react to the comfort constraints imposed by the users. Continuous devices implemented in this paper are heating and electric vehicles.

1) Wet Appliances: Wet appliances include washing machines, dishwashers and tumble dryers. Important characteristics of these devices are penetration rates, load cycles, start- and stop times and number of cycles per week. All these data for Belgium are taken from the smart-A project [19]. This data contains a set of 5000 load cycles for every type of device. According to the penetration rate, a household may or may not possess a certain type of device.

2) Heat Pump & Auxiliary Heaters: Each residence is equipped with an air coupled heat pump (HP) that can deliver both space heating (SH) and domestic hot water (DHW). To ensure that the temperature constraints set by the inhabitants can always be met, the HP is backed up with two auxiliary electric heaters AUX1 and AUX2. The building model and the DHW model are both based on [20].

a) Space Heating: The building model consists of one zone, heated by a floor heating system and is represented by an RC model based on [21]. Since this is a linear approach, the thermal behaviour of the building can be described by a linear state space model. The temperature in the houses is influenced by the incoming solar radiation and the ambient temperature. These data are obtained from [22]. Furthermore it can be adjusted by the use of the heat pump or the auxiliary heaters. Three types of newly built buildings are considered; two types of single family houses and one type of multi family house from [21] and [23].

b) DHW: The domestic hot water is supplied by and stored in a hot water storage tank. The thermal power that needs to be supplied to the hot water tank at each moment depends on the water extracted from the tank. It is assumed that the first two inhabitants use 50 liter per person per day and each next inhabitant consumes an extra 30 liter per day. Per person 2 to 3 tapping moments per day are assumed and the probability of DHW consumption over a day is based on Peuser et al. [24]. The water in the tank can only be heated by the heat pump and AUX1.

3) Electric Vehicles: Based on [25] and [26], it is assumed that 71.4 % of the households possesses an EV. The moments that EVs can charge depend on the driving behaviour. Data of driving behaviour is based on [27] and includes 100 different driving patterns for each day. Important parameters in these data include moments when the EV is at home, when it's driving and when it needs a certain amount of energy to drive a certain distance.

B. Optimization Problem

All devices are grouped into a house according to their penetration rate. This house receives a RTP and a central optimizer schedules the flexible devices towards minimal electricity cost. Because of the discrete nature of the wet appliances, this results in a mixed-integer linear problem (MILP). The constraints of this optimization problem are:

1) Wet Appliances: The load cycle of wet appliances can only be shifted between the start time and the stop time imposed by the user.

2) SH & DHW:

- The temperature must always lie between 16°C and 22°C when occupants are absent and between 20°C and 23°C when the house is occupied.
- Thermal power can only be consumed. The upper limit is set by the maximum equipment ratings for the different heaters.

3) EV: The state of charge of the battery has to be at least 20 %. Vehicle-to grid is not supported by the model. Consequently, power can only be consumed. The maximum charging power is limited by the household connection.

4) Overall Constraint: The total electric power that can be drawn by a dwelling is limited to 9.2 kVA as this is the standard for the average household [28].

5) Cyclical Constraints: The optimization problem has a finite horizon N. Start and end conditions have to be imposed. Therefore, extra cyclical constraints are imposed to all variables. By imposing these cyclical constraints, the influence of the first following time step $N+1$ is taken care of by the first time step. This is done for all temperatures: $T_1 = T_N$ and analogously for the state of charge (SOC) of the EV.

C. Neighbourhoods

Several parameters in the model of a house are stochastically defined. They are summarized in Table I. The number of occupants per household is inspired on the population structure of Belgium [29]. In order to represent the penetration rates and the number of occupants per household in a realistic way, multiple houses are created. Following Veldman [30], they are grouped together in neighbourhoods of 70 houses. The energy consumption of such a neighbourhood is optimized in the same way as described in Section II-B.

Table I
STOCHASTIC ELEMENTS IN THE MODEL

House	Number inhabitants	EV	Penetration
	Occupancy		Energy needed
	Non flexible loads		Time to charge
Wet appliances	Penetrations	DHW	Hot water demand
	Load cycle	SH	Type house
	Start & stop times	PV	Penetration & size

III. REGRESSION ANALYSIS

An optimization as performed in Section II leads to a certain electricity demand q given a price p for each time step. These optimizations are repeated for different price signals and lead to multiple (p, q) points. Given these points, a regression can be performed that yields coefficients that express the relationship between price p and demand q at each moment. In this paper, a linear relationship is proposed and the coefficients of the regression are assembled in a so called 'price elasticity matrix' $\epsilon_{N\times N}$. This matrix represents the relationship between a change in electricity price Δp and a change in demand Δq, relative to a certain reference point (p_{ref}, q_{ref}):

$$\epsilon_{N\times N} \cdot \Delta p = \Delta q, \qquad (1)$$

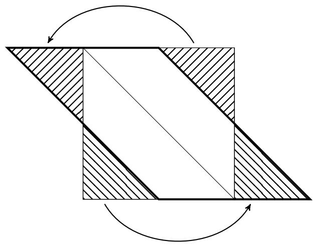

Figure 1. Effect of the cyclical constraints on the elasticity matrix

with

$$\epsilon_{N\times N} = \begin{pmatrix} \epsilon_{11} & \cdots & \epsilon_{1N} \\ \vdots & \ddots & \vdots \\ \epsilon_{N1} & \cdots & \epsilon_{NN} \end{pmatrix} \tag{2}$$

and:

$$\Delta q = \frac{q - q_{ref}}{q_{ref}}, \tag{3a}$$

$$\Delta p = \frac{p - p_{ref}}{p_{ref}}, \tag{3b}$$

the relative differences in electricity consumption and price, w.r.t. a reference point on an hourly basis. In this matrix, the own-price and cross-price elasticity are defined as:

$$\epsilon_{ii} \cdot \Delta p_i = \Delta q_i, \qquad \epsilon_{ij} \cdot \Delta p_j = \Delta q_i. \tag{4}$$

The elasticities thus define a linear relationship between the deviation of a price at a certain hour j from a reference price and the deviation of the associated electricity consumption at the hour i.

The time horizon N of prices that influence the electricity use is taken to be 24 as the typical cycle of residential electricity use repeats itself every 24 hours. Later on in this paper, it will be proven that this horizon is sufficient. Special attention has to be paid to the cyclical constraints set in Section II. These constraints make that the upper-right and the lower-left triangle of the elasticity matrix should be shifted as depicted in figure 1.

A. Data Processing

The price vectors p used in the optimizations are obtained from Belpex prices of 2012 and 2013 [31]. These prices are rescaled in order to all have the same mean value of 1. In order to not let them influence the regression, extreme situations or outliers — price vectors with a price more than 3 times the standard deviation of the mean value for that hour — are removed from the sample set. As the optimizations are computationally intensive, a compromise between computational time and the quality of the regression has been made. As a good compromise, $n = 350$ price vectors of 24h are retained.

B. Least Squares Estimate

To obtain the linear price elasticities, a linear regression is performed that relates a vector Δp of size N (24 in case of one day) to a vector Δq, also of size N. These are in fact N independent regressions for each $\Delta q_i, i = 1, ..., N$ if we assume that each element Δq_i only depends on Δp. Hence, N multiple linear regressions are performed. This means finding a vector $\boldsymbol{\epsilon_i} = (\epsilon_{i1}, \epsilon_{i2}, \ldots, \epsilon_{iN})$ such that:

$$\Delta q_i^k = \boldsymbol{\epsilon_i} \Delta p^k + err_i^k \tag{5}$$

with err_i^k the error on the linear model for time step i and sample k. If we define:

$$\boldsymbol{err_i} = (err_i^1, \ldots, err_i^n) \tag{6}$$

$$\boldsymbol{\Delta q_i} = (\Delta q_i^1, \Delta q_i^2, \ldots, \Delta q_i^n), \quad \forall i = 1, \ldots, N \tag{7}$$

and:

$$\boldsymbol{\Delta p} = \begin{pmatrix} \Delta p_1^1 & \cdots & \Delta p_1^n \\ \vdots & & \vdots \\ \Delta p_N^1 & \cdots & \Delta p_N^n \end{pmatrix} = (\Delta p^1, \ldots, \Delta p^n) \tag{8}$$

we have a linear multiple regression from $\mathbf{\Delta p}$ to $\mathbf{\Delta q_i}$, which can be completely summarized as:

$$\boldsymbol{\Delta q_i} = \boldsymbol{\epsilon_i \Delta p} + \boldsymbol{err_i}, \quad \forall i = 1, \ldots, N \tag{9}$$

An unbiased estimator $\boldsymbol{\epsilon_i}$ would be the least square estimator $\boldsymbol{\hat{\epsilon_i}}$:

$$\boldsymbol{\hat{\epsilon_i}} = (\boldsymbol{\Delta p}^T \boldsymbol{\Delta p})^{-1} \boldsymbol{\Delta p}^T \boldsymbol{\Delta q_i} \tag{10}$$

Since we made the prices relative to the mean price, the matrix $\boldsymbol{\Delta p}$ has only $N - 1$ independent rows. The classical least squares estimation cannot be applied to matrices less than full row rank N [32]. This problem can be overcome by using the Moore-Penrose pseudo inverse [33]:

$$\boldsymbol{\Delta p}^+ = V S^+ U^T \tag{11}$$

with $\boldsymbol{\Delta p} = USV^T$ the singular value decomposition of $\boldsymbol{\Delta p}$. The estimator $\boldsymbol{\hat{\epsilon_i}}$ is then:

$$\boldsymbol{\hat{\epsilon_i}} = \boldsymbol{\Delta p}^+ \boldsymbol{\Delta q_i} \tag{12}$$

Since this regression is performed without intercept, R^2 cannot be relied on as a measure for the quality of the fit [34]. A general test that can be performed to assess if the linear regression is qualitative, is the test of significance. This test indicates that the linear regression performed is indeed meaningful.

C. Selective Regression

Not every hour contributes as much to the resultant Δq_i. A better model would be obtained if the hours that do not contribute much to Δq_i could be excluded and only the hours that are significant are retained. This can be done formally by performing a *selective regression*, in which only the variables that are statistically significant are taken into account. This results in a reduced linear model $\epsilon^{sel}_{N\times N}$. Since the full matrix $\epsilon^{full}_{N\times N}$ from (12) contains all coefficients $\epsilon_{ij}, j = 1, \ldots, N$, the matrix $\epsilon^{sel}_{N\times N}$ can be thought of as a linear model 'nested' inside the full linear model $\epsilon^{full}_{N\times N}$.

To determine whether the full model really fits the data better than the nested model (the selective regression), a statistical *F-test* can be used. Let ϵ_{sub} be the vector of all

ϵ_{ij} that are zero in the nested model. The hypothesis test is expressed as:

$$H_0: \quad \epsilon_{sub} = 0 \qquad \text{against} \qquad H_1: \quad \epsilon_{sub} \neq 0 \qquad (13)$$

This results in an F-statistic with $p_2 - p_1$ and $n - p_2$ degrees of freedom [35]:

$$F = \frac{(SSerr_1 - SSerr_2)/(p_2 - p_1)}{SSerr_2/(n - p_2)} \qquad (14)$$

with $SSerr_1$ and $SSerr_2$ the sum of squares of the errors of the nested model 1 and the full model 2 respectively, p_1 and p_2 the number of coefficients of model 1 and 2, so that $p_2 > p_1$.

Own-price elasticities and the values centred around it are considered the most important. Since the objective of the selective regression is to find to which extent the elasticities are significant when stepping away from the diagonal, the procedure is carried out as follows. A first regression is performed taking only the own elasticity ϵ_{ii} into account. Then, the coefficient to the right $\epsilon_{i,i+1}$ is added, a new regression is performed and it is assessed whether or not the contribution of this coefficient is significant by (14). If this is the case, this model is extended with $\epsilon_{i,i+1}$. This procedure is then repeated for the coefficient to the left of the diagonal $\epsilon_{i,i-1}$, then again for $\epsilon_{i,i+2}$ and so on. This is repeated until it is assured that the selective model ϵ_i^{sel} is as good as the full model ϵ_i^{full} by performing (14) with the full model as reference.

D. Elasticity of Separate Devices

The influence of the three types of flexible devices will be discussed later in section V. It is possible to calculate the elasticity matrix $\epsilon_{N\times N}$ for each type of device separately as can be seen when considering following equation:

$$\begin{aligned}\epsilon_{N\times N} \cdot \Delta p &= \Delta q \\ &= \frac{q^{WA} - q^{WA}_{ref}}{q_{ref}} + \frac{q^{EV} - q^{EV}_{ref}}{q_{ref}} + \frac{q^{Heat} - q^{Heat}_{ref}}{q_{ref}} \\ &= \epsilon^{WA}_{N\times N} \cdot \Delta p + \epsilon^{EV}_{N\times N} \cdot \Delta p + \epsilon^{Heat}_{N\times N} \cdot \Delta p, \quad (15)\end{aligned}$$

where q^{WA}, q^{EV} and q^{Heat} is the electricity use of the wet appliances, EVs and heaters respectively. The elasticity matrix can thus be split up into 3 'sub-matrices', one for each device, which should sum up to the total matrix $\epsilon_{N\times N}$.

IV. Monte Carlo Simulation

A. Methodology

The Monte Carlo technique is essentially a methodology that uses sample means to estimate population means [36]. Consider the function $z(x)$, which depends on a stochastic variable x with a with a probability density function (PDF) $f(x)$. According to the Monte Carlo method, its population mean, or expected value can be approximated by the *sample mean* of z:

$$\bar{z} = \frac{1}{N_m} \sum_{i=1}^{N_m} z(x_i), \qquad (16)$$

where x_i are N_m randomly sampled values of the variable x according to its PDF $f(x)$. It can be shown that $\lim_{N_m\to\infty} \bar{z} = \langle z \rangle$.

The standard error $\sigma(\bar{z})$ of the estimate of $\langle z \rangle$ can be approximated by [37]:

$$\sigma(\bar{z}) \approx \frac{s(z)}{\sqrt{N_m}} \approx \sqrt{\frac{\overline{z^2} - \bar{z}^2}{N_m}}, \qquad (17)$$

with $\bar{z}$ as in (16), $s(z)$ the sample standard deviation and $\overline{z^2} = (1/N_m)\sum_{i=1}^{N_m} z(x_i)^2$. This standard error converges to zero with a rate $\sim \sqrt{1/N_m}$.

A Monte Carlo simulation can be seen as a stochastic simulation of some kind of model with a source of randomness. The result of a simulation is then a value $z(x_i)$, which is used in (16) to estimate the population mean $\langle z \rangle$.

B. Application

The model for flexible residential demand (see section II) contains a lot of stochastic elements, listed in table I. This means that each generated neighbourhood is different and so will also be the price elasticity matrix for a neighbourhood. The whole process can be thought of as a simulation with stochastic variables x. To be able to make conclusions for a whole region consisting of a lot of different neighbourhoods, the average behaviour of these neighbourhoods has to be known. This can be found by applying a Monte Carlo simulation on the model. The results from the simulation — the reference electricity use q_{ref} and the elasticity matrix $\epsilon_{N\times N}$ — are then the dependent values $z(x)$.

By performing the same simulation for different neighbourhoods, one can obtain different samples $z(x_i)$ of these values. Applying equation (16) on the samples gives a Monte Carlo estimate of the real average value. In order to obtain a good estimate, a sufficient amount of neighbourhoods has to be created. However, performing the regression on a neighbourhood is already computationally very expensive, so again a compromise has to be made. If we perform around $N_m = 100$ simulations then the standard error of the mean $\sigma(\bar{z}) \approx s(z)/\sqrt{100} = 0.10 \cdot s(z)$ is brought down to 10 % of its standard deviation.

V. Results

A Monte Carlo simulation is performed on 100 neighbourhoods for an average day of each season, both for weekdays and weekend days. The influence of the weather — mainly the ambient temperature and solar radiation — will become clear. The difference between weekdays and weekend days is made because people tend to be more at home in weekends, which might also have an impact on the flexibility.

A. Reference Scenario

As stated in section III, the elasticities are defined w.r.t. a certain reference point (p_{ref}, q_{ref}). The reference price p_{ref} is taken to be an average of all Belpex [31] prices, and shown in figure 2.

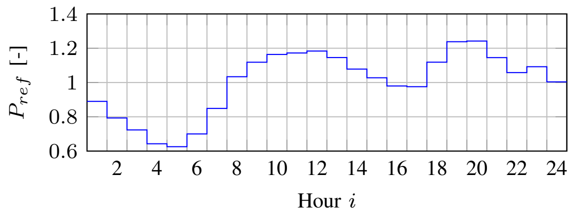

Figure 2. Reference price signal

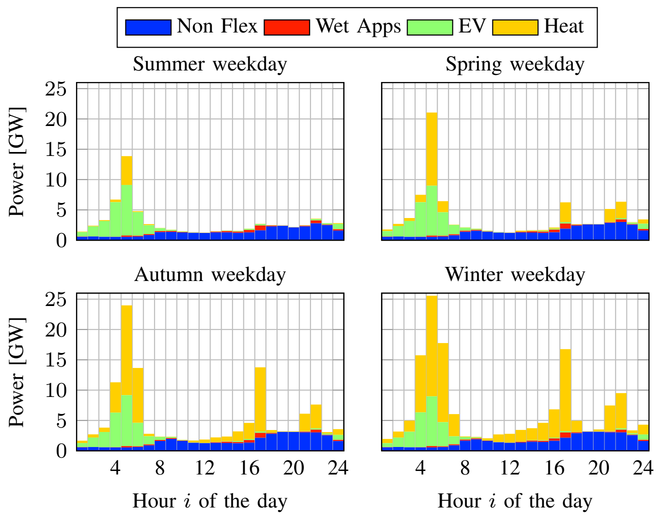

Figure 3. Reference residential electricity consumption scaled up to the country level of Belgium in summer, spring, autumn and winter.

The corresponding average electricity consumption is found by applying (16) on the electricity consumption of 100 neighbourhoods, when they receive the price signal p_{ref}. Since this is an estimate of the average over all possible neighbourhoods, these results can be scaled up to the country level of Belgium[1]. The resulting electricity consumption is given in figure 3, for the four different seasons on a weekday. The weekend days are not shown, as the differences are almost negligible.

1) Annual Electricity Consumption: From these figures one can calculate the average annual electricity consumption per appliance, which is shown in Table II. One can clearly see the big contribution of the heaters, especially in winter and autumn. It can also be noticed that almost all EVs are charged during the night, and that wet appliances contribute little, and mostly in the late afternoon.

The total electricity consumption is more than twice the amount of residential electricity consumption today, around 4 387 kWh/year per household. The doubling can be explained by the full electrification in the model. All heating is electric and fossil fuelled cars are replaced by EVs.

2) Peak Power Consumption: The hourly electricity demand when subject to the reference price is shown in figure 3. Although the total yearly electricity consumption will only double, the peak power demand in our reference scenario increases by a factor 5 to 8 when comparing to the situation

[1]This is done by multiplying by a factor 4 606 544/70, since 4 606 544 Belgian households are assumed and one neighbourhood consists out of 70 households

Table II
BREAKDOWN OF THE ANNUAL AVERAGE RESIDENTIAL ELECTRICITY CONSUMPTION IN THE MODEL, FOR ONE HOUSEHOLD SUBJECT TO THE REFERENCE PRICE.

Type of Demand	Energy consumption [kWh/year]
Non-Flexible	3 091
Electric Heating	3 587
Electric Vehicle	2 146
Wet appliances	304
Total	**9 128**

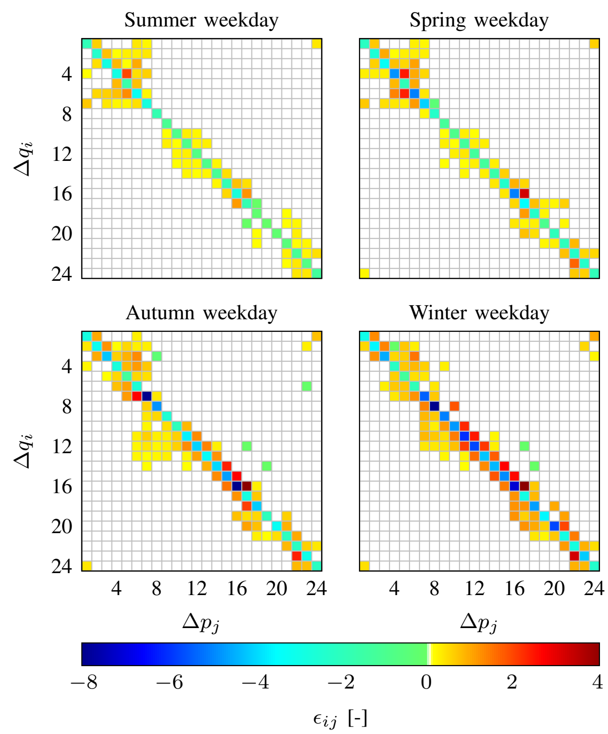

Figure 4. Values of the Monte Carlo estimates of the mean price elasticity matrices for a weekday in winter, autumn, spring and summer

today. This can be explained by the fact that an RTP strategy is used with automatic demand response, whereas currently there is only a day and night tariff. Due to this strategy, flexible demand will massively shift to the moments where electricity is cheapest, a phenomenon called *load syncing*.

The differences in peaks between the different seasons are mainly due to the heating. In every season a peak around hour 5 is noticed, which is due to both heating and EVs, while in spring, autumn and winter a second peak arises around hour 17, this time only due to heating.

B. Elasticities

The elasticity matrix, as defined by (1), is characteristic for each neighbourhood. Again, in order to get an estimate of the elasticity matrix independent of the specific neighbourhood investigated, a Monte Carlo simulation is performed on the elasticity values. Figure 4 shows a heat map of the resulting average elasticity matrices for an average weekday in all four seasons.

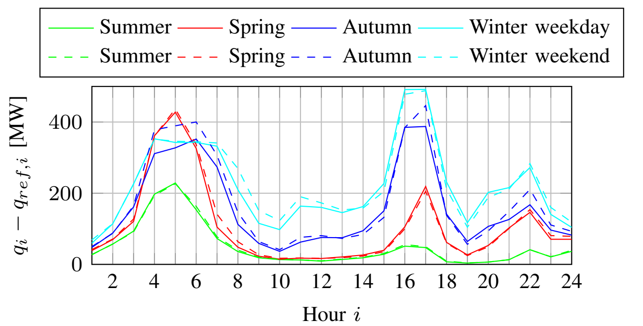

Figure 5. Absolute shift in electricity consumption $q_i - q_{ref,i}$ with a price decrease of $\Delta p_i = -1\%$, at every hour i and for the four seasons, on a weekday and a weekend day. The numbers are scaled up to a Belgian level.

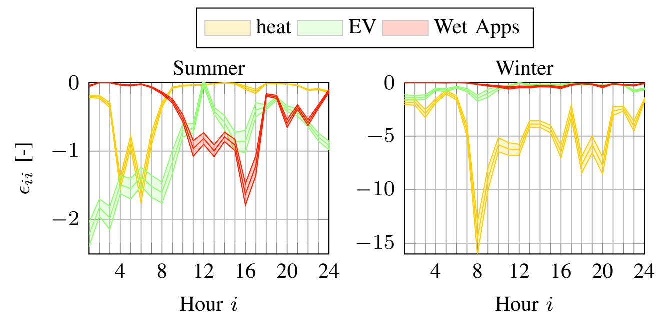

Figure 6. Own elasticities of the different devices (15), for a summer and winter weekday. The bands designate the 96% confidence interval.

One can clearly note the difference between diagonal values, which are the negative own elasticities, and the off-diagonal values or cross elasticities, which are almost all positive.

The most extreme values are found in winter, both positive and negative. This is caused by the heat pumps and electric heaters that are operating then and that provide a lot of flexibility. The smallest values are found in summer, since almost no space heating can be used here. The only available flexibility then is coming from electric vehicles and wet appliances. In spring and autumn, the values lie in between the ones in summer and winter, with in autumn greater values than in spring (positive and negative).

1) Own Elasticities: The own-price elasticities are the most negative at hour 7, 8 and hour 16, especially in winter and autumn. However, since they are depending on a reference electricity consumption $q_{ref,i}$, it is not immediately allowed to conclude that these are the hours where the most or the cheapest flexibility is available. By calculating the absolute shift $q_i - q_{ref,i}$ in electricity demand, one obtains an estimate that is independent of q_{ref}. This is shown in figure 5, for a relative decrease of 1% in the electricity price Δp_i at every hour i.

From this figure, one can see that there is little difference between a weekday and a weekend day. In all four seasons one can notice two main peaks: one during the night, around hour 5, and one in the afternoon, around hour 17.

In general, a clear trend can be seen: when temperatures and solar irradiance decrease, elasticities are bigger (in absolute value). For a same difference in electricity price, more electricity consumption will be shifted when temperatures are colder. Hence, it can be stated that more flexibility is available.

2) Cross Elasticities: The own-price elasticities are the biggest and thus the most influential. However, the cross elasticities are also important. They allow to determine how much and how far in time (a part of) the electricity consumption could be displaced. The furthest shift in demand in reaction to a price change is 7 hours, from hour 7 to hour 14 on an autumn weekday (not shown in figures).

It can be seen that in all seasons, generally electricity consumption can be displaced from the early morning (hour 4–6) to the first hours of the night (hour 1–4). Electricity consumption at hour 8 is almost only determined by its own-price elasticity, and a few cross-price elasticities in winter and autumn, meaning that not much electricity can be shifted to this point in time. There are a lot of cross-price elasticity values to the left of hours 10–14, meaning that it is possible to shift consumption to these hours from the earlier morning.

3) Influence of Different Devices: The influence of the three different types of devices can be determined by calculating their separate elasticities according to (15). Figure 6 presents these own elasticities for summer and winter. One can see that the heaters determine the flexibility almost completely during winter, although in summer they are only a little significant in the night. The EVs contribute during the night, both in summer as in winter, but during the day their elasticity is almost zero. The wet appliances have a limited influence during the day. The large negative peak in winter around hour 8 is mostly due to the low value of the reference electricity consumption at that moment.

VI. Conclusion

This paper presented a method to quantify the flexibility of automatic demand response applied to residential electricity demand using price elasticities. This is applied to a full-electric scenario in 2050 in Belgium.

The results show that the annual residential electricity consumption doubles w.r.t. the one we experience today. The peak power demand increases by a factor 5 to 8. This is due to the electrification of residential energy and the RTP strategy.

The resulting price elasticities (and thus flexibility) are strongly influenced by weather conditions. A higher flexibility is available in winter (because more heating appliances are available), the least flexibility is available in summer. There is little difference between weekdays and weekends.

The elasticity values are quite large, meaning that a RTP scheme is very sensitive to price changes.

The maximum shift in time of (a part of) the electricity demand to another hour is found to be 7 hours.

Heaters provide the most flexibility in each season but the summer. EVs are mainly available for load shifting during the night and wet appliances only contribute little to the overall flexibility during the day.

References

[1] European Climate Foundation, "Roadmap 2050: Practical guide to a prosperous, low-carbon Europe," ECF, Tech. Rep., 2011.

[2] COM(2011) 885/2 from the European Commission, "Energy roadmap 2050," 2011.
[3] COM(2011) 112 from the European Commission, "A roadmap for moving to a competitive low-carbon economy in 2050," 2011.
[4] Eurelectric, "Power choices: Pathways to carbon-neutral electricity in Europe by 2050," Eurelectric, Tech. Rep., 2011.
[5] S. Teske, J. Muth, S. Sawyer, T. Pregger, S. Simon, T. Naegler, M. O'Sullivan, S. Schmid, J. Pagenkopf, B. Frieske *et al.*, "Energy [r]evolution: a sustainable world energy outlook," Greenpeace International, EREC and GWEC, Tech. Rep., 2012.
[6] M. Albadi and E. El-Saadany, "Demand response in electricity markets: An overview," in *IEEE Power Engineering Society General Meeting*, vol. 2007, 2007, pp. 1–5.
[7] Edward J. Bloustein School of Planning and Public Policy, "Assessment of customer response to real time pricing," Rutgers - The State University of New Jersey, Tech. Rep., June 2005.
[8] C. Nabe and G. Papaefthymiou, "All island renewable grid study updated to include demand side management," Ecofys Germany GmbH, Tech. Rep., March 2009.
[9] D. Devogelaer, J. Duerinck, D. Gusbin, Y. Marenne, W. Nijs, M. Orsini, and M. Pairon, "Towards 100 % renewable energy in Belgium by 2050," VITO, ICEDD, Federal Planning Bureau, Tech. Rep., April 2013.
[10] T. Lambert, P. Gilman, and P. Lilienthal, *Micropower System Modeling with Homer*. John Wiley and Sons, 2006.
[11] S. Koch, J. L. Mathieu, and D. S. Callaway, "Modeling and control of aggregated heterogeneous thermostatically controlled loads for ancillary services," in *Proc. PSCC*, 2011, pp. 1–7.
[12] W. Zhang, J. Lian, C.-Y. Chang, K. Kalsi, and Y. Sun, "Reduced-order modeling of aggregated thermostatic loads with demand response," *51st IEEE Conference on Decision and Control (CDC)*, pp. 5592–5597, Dec. 2012. [Online]. Available: http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=6426010
[13] M. Alizadeh, A. Scaglione, and R. J. Thomas, "From packet to power switching: Digital direct load scheduling," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 6, pp. 1027–1036, 2012.
[14] N. Leemput, F. Geth, B. Claessens, J. Van Roy, R. Ponnette, and J. Driesen, "A case study of coordinated electric vehicle charging for peak shaving on a low voltage grid," in *3rd IEEE PES International Conference and Exhibition on Innovative Smart Grid Technologies (ISGT Europe)*. IEEE, 2012, pp. 1–7.
[15] C. De Jonghe, "Short-term demand response in electricity generation planning and scheduling," Ph.D. dissertation, KU Leuven, 2011.
[16] A. Faruqui and S. Sergici, "Household response to dynamic pricing of electricity: a survey of 15 experiments," *Journal of Regulatory Economics*, vol. 38, no. 2, pp. 193–225, 2010.
[17] A. Scaglione, "Networks and Markets for Scheduling Energy Consumption," 2013, unpublished presentation.
[18] I. Richardson, M. Thomson, D. Infield, and A. Delahunty, "A modelling framework for the study of highly distributed power systems and demand side management," in *International Conference on Sustainable Power Generation and Supply*, April 2009, pp. 1–6.
[19] R. Stamminger and G. Broil, "Synergy potential of smart appliances," Smart-A, Tech. Rep., 2008.
[20] K. Bruninx, D. Patteeuw, E. Delarue, L. Helsen, and W. D'haeseleer, "Short-term demand response of flexible electric heating systems: the need for integrated simulations," in *10th International Conference on the European Energy Market (EEM)*. IEEE, 2013, pp. 1–10.
[21] T. Van Oevelen, "Regeling van warmtepompsystemen in woningen, implementatie van modelgebaseerde predictieve regeling (in Dutch)," Master's thesis, KU Leuven, 2008.
[22] "Meteonorm version 6.1 - edition 2009," 2009.
[23] TABULA, "Residential building typology," Online: http://www.building-typology.eu/building-typology/country/be/, accessed: 2014-06-03.
[24] F. A. Peuser, K.-H. Remmers, and M. Schnauss, *Solar thermal systems: Successful planning and construction*. Beuth Verlag GmbH, 2002.
[25] N. Tanaka *et al.*, "Technology roadmap: Electric and plug-in hybrid electric vehicles," *International Energy Agency, Tech. Rep*, 2011.
[26] D. Schalck *et al.*, "Mobiliteitsrapport van Vlaanderen," Mobiliteitsraad van Vlaanderen, Tech. Rep., 2009.
[27] J. Van Roy, N. Leemput, S. De Breucker, F. Geth, P. Tant, and J. Driesen, "An Availability Analysis and Energy Consumption Model for a Flemish Fleet of Electric Vehicles," in *European Electric Vehicle congress (EEVC)*, October 2011.
[28] VREG, "Vlaamse regulator van de elektriciteits- en gasmarkt," Online: http://www.vreg.be/eenvoudige-aansluiting/, accessed: 2014-06-03.
[29] FOD Economie Belgium, "Structuur van de bevolking volgens huishoudens: per jaar, gewest en grootte," Online: http://statbel.fgov.be/nl/statistieken/cijfers/bevolking/structuur/huishoudens/jaar_gewest_grootte/, accessed: 2014-06-03.
[30] E. Veldman, "Impacts of flexibility in future residential electricity demand on distribution network utilisation," Ph.D. dissertation, Eindhoven University of Technology, September 2013.
[31] Belpex, "The Belgian power exchange," http://www.belpex.be/, accessed: 2014-06-03.
[32] L. Fahrmeir, K. Thomas, S. Lang, and B. Marx, *Regression. Models, Methods and Applications*. Springer, 2013.
[33] M. Diehl, *Script for Numerical Optimization Course*. KU Leuven, 2012.
[34] S. Weisberg, *Applied linear regression*. John Wiley & Sons, 2005, vol. 528.
[35] "Graphpad curve fitting guide," Online: http://www.graphpad.com/guides/prism/6/curve-fitting/index.htm?reg_approaches_to_comparing_models.htm, accessed: 2014-05-09.
[36] W. L. Dunn and J. K. Shultis, *Exploring Monte Carlo Methods*. Elsevier, 2011.
[37] C. Lemieux, *Monte Carlo and Quasi-Monte Carlo Sampling*. Springer, 2009, vol. 20.