FACULTEIT ECONOMIE EN BEDRIJFSKUNDE

PENSION REFORM IN A GENERAL EQUILIBRIUM OLG MODEL

LESSONS FROM JAPAN

Word count: 18 138

Thomas Lebbe

Student number : 00905327

Supervisor: Prof. dr. Freddy Heylen

Master's Dissertation submitted to obtain the degree of:

Master of Science in Economics

Academic year: 2018 - 2019

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Brugge, 12/08/2019

Thomas Lebbe

Acknowledgments (in Dutch)

Woorden schieten tekort en waarschijnlijk vergeet ik tal van mensen zonder wiens steun ik nooit had kunnen afstuderen.

Eerst wil ik mijn ouders bedanken. Zonder jullie zou ik niet zijn wie ik vandaag ben. Ik ben jullie zoveel schuldig dat ik niet kan omschrijven hoe dankbaar ik ben.

Ik wil ook mijn promotor Prof. Dr. Freddy Heylen bedanken voor zijn passie en wijze raad. Professor, uw lessen zijn een inspiratie. Zonder u zou mijn liefde voor macro-economie niet bestaan hebben. Ook bedankt voor de kansen die u me geeft, ik apprecieer het enorm.

Prof. Dr. Tim Buyse wil ik ook bedanken. U heeft veel moeite gestoken in het begeleiden van de les gevorderde macro-economie. Ook was u altijd flexibel wanneer mijn werk het lessenrooster in de war stuurde. Bedankt voor uw toewijding en begrip.

Dank ook aan Pieter en Willem van de vakgroep, zonder jullie kalmte en hulp op lastige momenten was ik er nooit geraakt.

Ten slotte wil ik ook mijn vrienden bedanken, in eerste plaats de *Bond Boys*, Ik hou van jullie. Maar ook de *Eco Boys*, uit diverse achtergronden en toch één grote bende. Merci gasten.

Brugge, Augustus 2019

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Abbreviations

CD	Cobb-Douglas
CES	Constant Elasticity of Substitution
ed.	Edition
Fig	Figure
FOC	First Order Condition
GDP	Gross Domestic Product
I.Q.	Intelligence Quotient
LHS	Left-Hand Side
NIPSSR	National Institute of Population and Social Security Research
OECD	Organization for Economic Cooperation and Development
OLG	Overlapping Generations
PAYG	Pay-As-You-Go
PISA	Programme for International Student Assessment
RHS	Right-Hand Side
TFR	Total Fertility Rate
WWII	World War II

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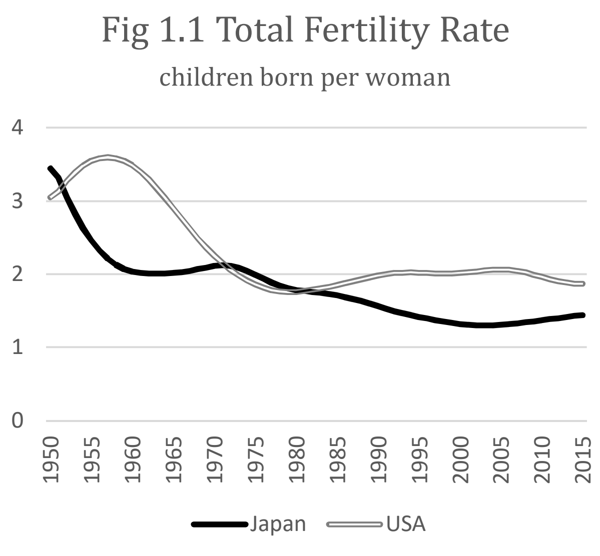
"*Responsibility… Given my abilities, what will be my response?*"

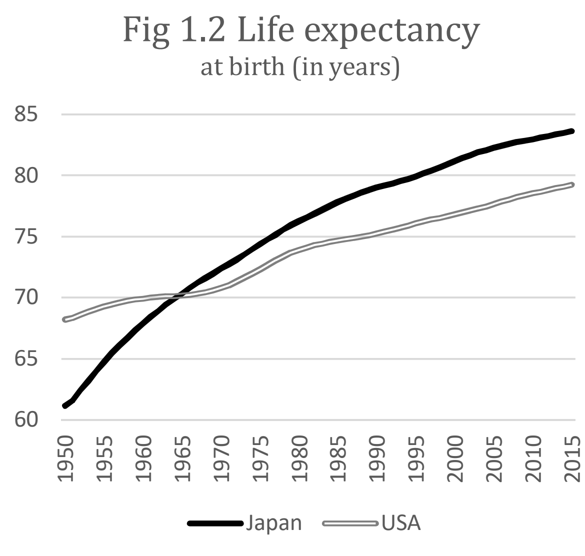
- Richard Pimentel
Disability rights activist

1 Introduction

1.1 Fertility and life expectancy

Most OECD countries are facing a demographic shift (OECD, 2016). Demographic tendencies are usually described by two main drivers: the fertility rate and the life expectancy. The demographic shift in most OECD countries is two-fold: on the one side the fertility rate has decreased and on the other side life expectancy has increased. Compared to other countries Japan has had a faster decline in fertility rate and a faster increase in life expectancy. This is shown in the figures below that compare the total fertility rate (figure 1.1) and the life expectancy (figure 1.2) for both the United States of America and Japan.





Source fig 1.1 and 1.2: United Nations, Department of Economic and Social Affairs, Population Division (2017). World Population Prospects: The 2017 Revision.

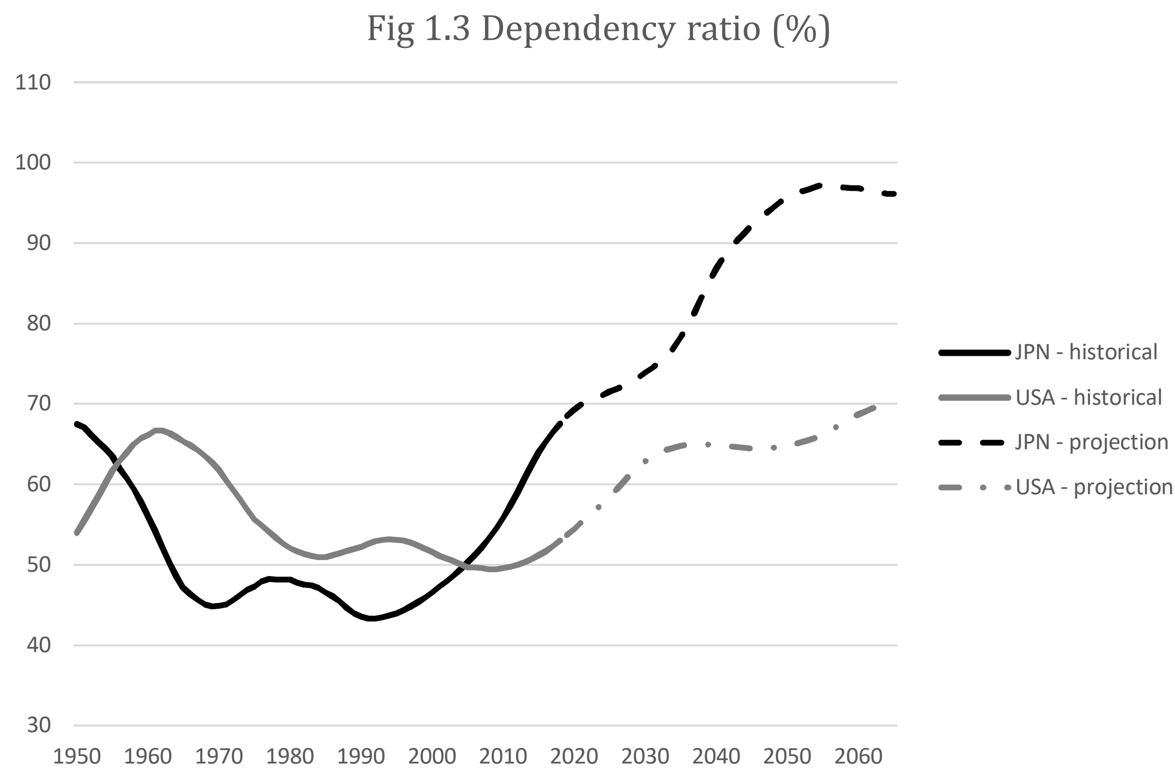
In macroeconomic terms this shift has some interesting consequences. First, it implies that we are dealing with a *negative arithmetic effect* that will hamper per capita growth if agents don't change behaviour. If life expectancy continues to increase and the fertility rates continue to decrease, the combination of fewer workers and a larger fraction of older - and hence dependent - people will lead to a decrease in per capita output (Onder and Pestieau, 2014). Added to this, longevity alters the composition of society. Generally speaking older cohorts are net dissavers (Goodhart and Erfurth, 2014). When their proportion in society increases this has an negative impact on aggregate savings, which on their turnare essential to investments. A negative impact on economic growth is thus expected with increased longevity.

Fortunately the demographic shift will also change the behaviour of individuals. Having a longer life prospect might induce the young to save more for their old age and thus increase aggregate savings (Krueger and Ludwig, 2007; Onder and Pestieau, 2014).

From the previous discussion it becomes clear that behavioural change in aggregate savings can counteract some of the effects of a demographic shift. Still, it remains unclear if it can neutralize all of the effects. With changing demographic tendencies, governments increasingly need to assess optimal policies to safeguard future economic growth to ensure the viability of their pension system.

1.2 Japan

Japan will be taken as a case study. Why Japan? Japan has proved to be interesting for several reasons. First, the country has been one of the earliest states to undergo this demographic shift (OECD, 2016). Compared to other OECD countries, it has undergone an increase in longevity and a decrease in fertility about 20 years before other countries. The shift is also quite large as can be deduced from the dependency ration in the figure below (Figure 1.3). Furthermore, in 2065 the population of Japan will reach half of the size it had in 2015. This clearly marks the magnitude of the Japanese demographic shift. (NIPSSR, 2017).



Data and projection from Roser and Ortiz-Ospina (2019).

The dependency ratio has been calculated as:

$$Dependency\ ratio = \frac{Population\ under\ 15 + Population\ \ 65\ and\ over}{Population\ between\ 15\ to\ 64}$$

The equation for the dependency ratio shows how large the non-active population is in comparison to the active population. Figure 1.3 makes clear that if no change in demographic behaviour occurs, per capita growth will face a hit.
The case of Japan can serve as an example for other large economies dealing with a demographic shift. Put bluntly, Japan today is what the USA might be in 20 years from now.

Another reason for choosing Japan is the government's recent decision of implementing a Bonus-Malus system for retirement[1] to encourage behavioural change: it needs to serve as a stimulus for delaying the retirement age. But the effects of this policy are not clear yet. Since most workers will already be entitled to the bonus at the current retirement age, the income effect might dominate over the substitution effect, which implies the risk of having introduced a very costly, yet ineffective policy.

A last reason for taking Japan as a case study, is the absence of migration towards Japan. In 2010 foreign residents constituted under 1.7% of the total population of Japan[2] and the net migration rate was 0.4% (OECD, 2018). In terms of demographic dynamics this becomes negligible.
As migration can be seen as the third driver of population dynamics (OECD, 2016), taking the case of Japan provides the opportunity to create a model which only takes into account the first two drives of population change: fertility and longevity. Hence it becomes possible to isolate those two drivers.

1.3 Overlapping Generations

Whilst there are different ways in evaluating behavioural change, overlapping generations models (OLG) seem an adequate way to evaluate the macroeconomic effects of a declining fertility and an increasing life expectancy. Changes do not occur in vacuum and as such a general equilibrium approach is desirable. Declining fertility and increasing life expectancy will affect the incentives for households and firms to work, save and invest. Because optimal behaviour changes over the life-cycle, it becomes important to also calculate the net effect when a shift occurs. This net effect is taken into account in an overlapping generation model. But what does an OLG model entail?

[1] See also IMF(2012) and OECD (2016).
[2] Due to very strict migration policies, one can assume migration to remain low in the near future. See also OECD (2016).

An OLG model consists of agents who divide up their lifetime in different periods. Per period a new generation is born and the oldest generation dies (de la Croix & Michel, 2002). This implies that at each moment multiple generations live together, and form “Overlapping Generations”, as the name of the model implies. The Model starts from the hypothesis that individuals have (perfect) foresight and can rightly predict future outcomes of their actions. Young individuals come into the model as adults and optimally decide how much time they want to allocate to work and how much they consume. What is not consumed by the individuals is saved. The choice between consuming and saving is done while taking the future into account: Saving implies additional wealth that can be used for future consumption.

In the model firms also optimize behaviour. They attract labour and capital by paying the individuals wages and rents. Firms produce the output that households consume and that other firms invest. The last actor in the model is the government, that levies taxes and that consumes goods. Choices made by the government impact the incentives of households and firms. Which in their turn will impact the macroeconomic outcome of society.

OLG models serve as a tool to assess effectiveness of policies, while taking into account the generational differences. As de la Croix & Michel (2002) put it, it is therefore an essential tool for the modern macroeconomist.

1.4 Improvements to the basic OLG model

Whilst the basic OLG model has its merits, it also has its shortcomings. The main critique against it is the homogeneity of agents within the same generation. Indeed, it seems unlikely that agents only differ in age but not in ability. To counterbalance this critique of the basic OLG model, authors like Sommacal (2006) and Fehr et al. (2013) have introduced heterogenous agents, which differ in their levels of human capital. Other authors - like Docquier and Paddison (2003)- take a different approach: agents start with the same initial level of human wealth but have different learning abilities that will alter their human capital stock. Lastly, authors like Buyse et al (2017) combine both adaptations to the model.

This dissertation follows the last group where individuals differ with respect to initial human capital stock and learning capacities.

Another improvement to the OLG Model is made by Buyse et al. (2013, 2017) in which the authors endogenize the retirement choice of agents. In earlier models, the retirement age was fixed. I too will endogenize the retirement choice and add to it a bonus-malus. The reason for choosing an endogenous retirement decision is that it reflects societal change better and adds additional dynamics to the model. The reason for choosing a bonus-malus system is that it reflects a policy that has been undertaken in Japan and is under discussion amongst academics in other countries[3].

1.5 Objectives

In this dissertation I will focus on the long-term effects of pension reform in a dynamic general equilibrium, modelled through an extensive OLG model. The aim will be to find optimal pension reform for a large closed economy in general and Japan in particular. Optimal pension reform will be assessed by three criteria: (i) macroeconomic outcomes, (ii) pension viability and (iii) equity. The focus lies on long-term growth for an economy confronted with a burden caused by an ageing population with low fertility.

To counterbalance the criticisms to the OLG model stated earlier, this dissertation will introduce heterogenous agents, which enrich the model. The relative new policy undertaken by the Japanese government, a Bonus-Malus system in pension payments, will also be incorporated in this dissertation. Muto et al. (2012) and Auerbach et al. (1989) have already done research on ageing in Japan through an OLG model. Yet research on long-run effects of the latest pension reform are lacking. In both cases the bonus-malus system and heterogenous agents are missing. This dissertation serves as a first step into assessing the effectiveness by focussing on the Bonus-Malus system of late/early retirement.

[3] E.g. the *Pensioen Commissie* (2013) in Belgium.

Effectiveness will be measured in two ways. Firslyt, it will be evaluated whether the policy has positive macroeconomic outcomes and secondly, it will be questioned whether policy can increase equity amongst high and low educated individuals and between the individuals of different generations. Concerning the macroeconomic outcomes, the focus will lay on retirement choice, hours worked, educational attainment and pension financing.

In order to measure the effectiveness this dissertation proceeds in three steps. First, an OLG model will be set up that can predict employment and growth for Japan. Secondly, demographic changes will be introduced in the model. Japan's latest pension reform will also be taken into account. Lastly, other policies that might outperform the bonus-malus system will be evaluated.

Even though this dissertation focuses on Japan, other economies will undergo the same ageing dynamics in the coming decade. It is the aim of this dissertation to contribute to the discussion being held in academia: for all countries are undergoing demographic changes that challenge their pension system.

1.6 Set-up

The set-up of the dissertation is as follows:

In Chapter 2, the build-up of the model is discussed and the different aspects inherent to the OLG model are explained. The levels of employment, retirement and education are compared between Japan and a few other OECD countries.

In Chapter 3, the equations of the model are presented. These equations are the technical equivalent of the model presented in Chapter 2.

The determination of the parameters are presented in Chapter 4. These parameters are determined by both literature and calibration.

The validity of the model is tested under Chapter 5. This is done by comparing the predictions to the real data. A counterfactual analysis of demography in the model is also presented in order to ensure the non-redundancy of demographics.

Once the model's validity has been tested, different policy options can be assessed. This will constitute Chapter 6.

Shortcomings and possible extensions to the approach taken into this dissertation will be the topic of Chapter 7. A conclusion is given in Chapter 8.

2 Specification of the Model

2.1 Specifications

This dissertation uses OLG as it was first modelled by Samuelson (1958) and Diamond (1965). Both the finite life and the life-cycle properties are already found in Diamond's work are present in the current model as well. Much adaptations have been introduced to the model since Diamond (1965): it has been extended from a two-period life to a multi-period life; and it has been enriched by the introduction of heterogenous agents as can be found in the work of Heylen & Van de Kerckhove (2019), and also Devriendt & Heylen (forthcoming) and Buyse et al. (2017).

Following their footsteps, this dissertation constructs an OLG model with 8 periods and includes heterogenous agents of different abilities. This yields 24 different types of agents at all time[4]. Each period lasts 10 years. Agents are born into the model at the age of 20 and die at the latest at the age of 100. Demographic change will affect the number of new-borns, and the fraction of agents who reach the higher ages. Each agent either born as low, medium or highly able. Their level of ability is linked to their initial level of human capital.

[4] Three abilities: low, medium and high over 8 periods in life, yields 24 different age-ability agents.

Depending on their ability, the youngest cohort (age 20-29) is able to pursue tertiary education. In the model this translates as follows: Low ability individuals do not study, whilst medium and high ability individuals can spend a fraction of their time on education. The more individuals study, the more they become productive in future periods. Which also increases their labour income.

Aside from education, individuals can spend time on work (age 20-69). For the fraction that they work, they receive wages. With those wages, agents can consume goods or save. What is saved, will yield additional income through interests which are payed in the next period. When they do not work, they enjoy leisure, which increases their utility. It becomes clear that all agents must weigh the costs and benefits of working and saving versus the pleasures of leisure and consumption.

During their last period of active life (age 60-69) agents chose when to retire. They can retire before the official retirement age of 65, but in doing so they lose some of their pension benefits. If they postpone their retirement to the age of 65 or later, the government rewards them by increasing their pension benefits.

In the last three periods of life (age 70-99) agents only enjoy leisure[5]. The only choice left to the individuals belonging to this age group is between saving and consuming.

[5] For a visual representation of all time constraints see *Figure 3.1 Life cycle and time constraint* in Chapter 3.

In the presented model several assumptions are made:
Firstly, parents with high abilities will only have high ability children. The same holds for low and medium ability parents and their children.

A second assumption in the model is that children do not inherit the stock of human capital from their parents. If the stock of human capital would transfer from generation to generation, one would enter a model of endogenous growth based on human capital accumulation, which goes beyond the scope of this dissertation[6].

A third assumption relates to the firms. Firms are supposed to act competitively on the labour, capital and output market. They produce by hiring capital and labour and by paying their marginal productivity. Firms are identical and produce one homogenous good. Free entry and perfect competition will assure zero profits.

The fourth assumption is the functioning of the government. The government levies taxes on consumption, capital and labour. It uses its revenues for buying goods and services, paying interests on public debt, for paying pension benefits and lump-sum transfers. When revenues and expenditures do not match, a surplus or a deficit is created and this changes the level of debt that the government has.

Finally, the OLG model is considered a closed economy. That means that both capital and labour are internationally immobile. As a result wages and rents are determined internally. On the labour market demand and supply equalize to determine the wage rate. The capital market also clears to determine the interest rate.

[6] See Buyse et al (2017) for an endogenous growth model based on human capital accumulation.

2.2 Comparison within OECD

A clear understanding of Japan's current situation requires a comparison with different countries. Some key variables that drive output are compared below between several OECD countries. The considered variables are the employment rate by age group, the rate of education and the age of retirement. As can be deduced from the table below which compares the above variables, Japanese citizens work a lot compared to other OECD member states. The work culture in Japan is reflected in working hours that exceed the 40 hours week[7].

Table 2.1: employment rate by age group, tertiary education (e) and effective retirement age (R): a comparison between Japan and other OECD members

	YOUNG (20-34)	MIDDLE (35-49)	OLDER (50-64)	e (YOUNG)	R
Belgium	40.88	45.44	23.44	14.1	57.9
Canada	48.72	55.6	40.32	13.6	62.1
Germany	39.76	44.16	27.92	17.2	61.1
USA	52.48	59.36	47.68	12.8	64.2
UK	48.64	54.72	39.52	12.3	62
Japan	53.94	57.81	36.11	20.7	68.1

Numbers from Heylen & Van de Kerckhove (2013) and Ministry of Internal affairs and communications (2017). All numbers are per capita annual hours in percentage of 2080 annual potential hours. With the exception of R, which represents the age of retirement.

In the above table the case study country Japan is compared to two of the core EU countries (Belgium and Germany) and to the Anglo-Saxon group of the OECD. This last group tends to score high on employment, but Japan clearly does not score badly in this regard. When looking at the retirement age, one concludes that the effective retirement age at 68 is higher in Japan than any of the other countries considered in the above table.

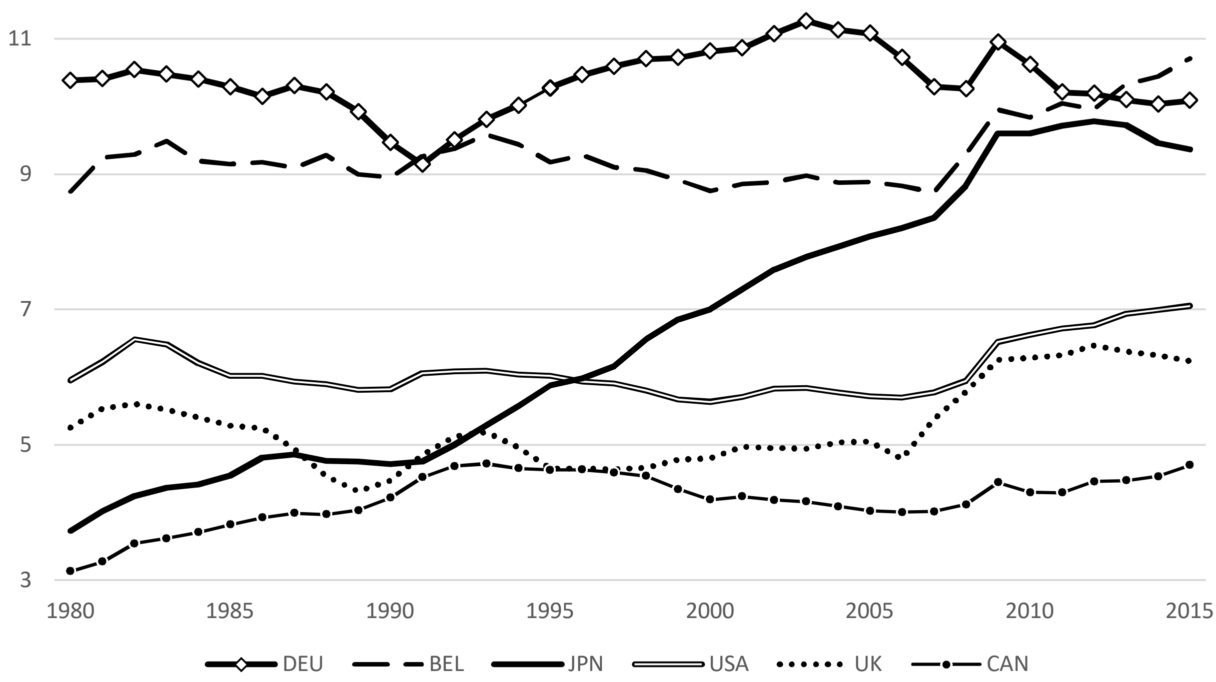
One could ask whether room for growth exists. And one can approve: Looking at the oldest category of workers one sees that the Japanese reduce their hours of work much more compared to most of the other countries. If only focussing on the group aged 60+, the employment rate in hours would

[7] Whilst regulation limits the workweek to 40 hours, many employees work overtime. For more details on hours worked per week see the Labour Force Survey (2017) Table 2-9-2 from the Statistics Bureau, Ministry of Internal Affairs and Communication of Japan. (via e-stat.co.jp)

drop further to 27%.

As a conclusion one can state that older workers reduce their labour supply rather than taking an early retirement. In this regard, Japan's recent policy initiative to reward late retirement, seems a meagre policy choice at best. As will be shown later, other policy initiatives might serve the cause of activating the older generation of workers better.

Fig. 2.1 Public pension as % of GDP



Source: OECD, Society at glance (2019)

In studying the financial aspects of pension systems, it is also important to examine how public pensions evolve over time. In figure 2.1 one clearly sees how Japan has had a big increase in public pension spending compared to other countries. As noted before, the demographic shift of Japan has preceded many other OECD countries.

An increase in pension payments can be expected in other economies that are currently still facing the demographic transition to an older population. This implies that the model used for pension reform and the policy under investigation should include demographics and take pension finances into consideration.

3 The model

In this Chapter the technical specifications of the OLG model are explained as well as the equations that go behind the model. The chapter begins with a general note on how time is modelled. Next, demographics, followed by households, firms and governments will be covered one by one. The chapter concludes by discussing the equilibrium

Overall, the model used in this dissertation follows the work of Devriendt & Heylen (forthcoming) and incorporates elements of the work of Buyse et al. (2013), whilst adding an endogenous retirement choice with a bonus-malus for late and early retirement.

3.1 Time in the model

Time is not continuous, but is modelled in discrete periods. Each period represents 10 years in real life. As a consequence, choices of agents are also discrete over time. Agents can only react to changes once every 10 years. This also applies to the rest of the model, such that all the variables are stated over 10 year periods.

Because agents enter the model at the age of 20 and die at the latest at the age of 100, they have 8 periods (at most) in which they can act. Depending on the period in life agents will face different choices.

3.2 Demographics

An essential part of the presented model is demographic change: The youngest cohort at time t relates to the youngest cohort at time t-1. To model this, one can follow the work of de la Croix (2013), Mérette & Georges (2009) and Devriendt & Heylen (forthcoming). In the equation below, f_t is the fertility rate which is compared between the youngest cohort at time t compared to the same age group one period earlier. This fertility rate exogenously enters into the model as follows:

$$N_1^t = f_t N_1^{t-1} \tag{1}$$

Where $f_t > 0$

The superscript t denotes that the cohort is born at time t, we call this cohort of generation t. The subscript j denotes its age. Equation (1) shows that size of the youngest cohort at time t relates to the size of the second youngest cohort but one period back. It becomes clear that this approach is different than the traditional use of the term fertility rate[8].

A single generation changes in size during its life cycle. Indeed, a fraction of that generation dies each period. Model-wise, this can be translated as such: each generation faces a probability sr_j^t that they will survive from age j to age j+1, on the condition of having reached age j. These survival rates are exogenously put into the model:

$$N_{j+1}^t = sr_j^t N_j^t \tag{2}$$

[8] Common use of fertility rate is children per woman. Model-wise this cannot be since the cohorts are only separated by 10 years in age. Alternatively I could have stated that generation t are the parents of generation t+3, in terms of the model this alternative changes nothing substantial.

Put differently, agents are born into the model at age 20 and have a probability of not surviving to the age of 30. When an agents reaches the age of 30, they have a probability of not surviving to the age of 40, and so on. The probability of surviving depends on the current age of the agent. Earlier in life the survival rates are higher than later in life. These probabilities of surviving are shown in eq. (2). Those agents that do reach the age of 90 have a zero probability of reaching the age of 100. Model-wise all agents die at the age of 100 at the latest. In the model this is symbolised by $0 < sr_j^t < 1$ and $sr_8^t = 0$.

Whenever a subscript "s" is added, e.g. $N_{j,s}^t$, this denotes the ability s. The different symbols for ability are: L for Low, M for Medium or H for High.

As stated before, the assumption made in this model is that the level of ability (low, medium or high) is passed on to the next generation. The total population at any time exists out of three ability groups:

$$N_j^t = N_{j,L}^t + N_{j,M}^t + N_{j,H}^t \tag{3}$$

Each having a fixed weight relative to the total population: v_L , v_M and v_H.
Making $v_L + v_M + v_H = 1$, such that: $N_j^t = (v_L + v_M + v_H)N_j^t \quad \forall j$

This implies that fertility and conditional survival rates are equal amongst all ability types, hence the lack of ability subscripts for the fertility and survival rates.

3.3 Households

3.3.1 Utility

Households maximize their expected lifetime utility as described in eq. (4). As agents can live up to 8 periods, they will sum the instantaneous utility of each of these 8 periods. However, agents will discount future utility for the probability of dying (π_j^t) and by discounting for the time preference (β). It is important to note that the combination of changing survival rates and the pure time preference changes the effective discount rate that households use. Consequently, the savings dynamics is altered when life expectancy increases.

$$U_S^t = \sum_{j=1}^{8} \beta^{j-1} \pi_j^t U(c_{j,s}^t, l_{j,s}^t) \tag{4}$$

Where β denotes the time preference ($0 < \beta < 1$), which is ability and age independent, and π_j^t denotes the unconditional probability to survive until age j. ($0 < \pi_j^t < 1$). It becomes clear that the link with the conditional survival rates is as follows[9]:

$$\pi_j^t = \prod_{i=1}^{j-1} sr_i^t$$

Within each period the instantaneous utility is:

$$U\left(c_{j,s}^t, l_{j,s}^t\right) = \ln\left(c_{j,s}^t\right) + \gamma_j \frac{{l_{j,s}^t}^{1-\theta}}{1-\theta} \tag{5}$$

Where consumption is in log-utility and leisure is in CES utility. The intertemporal elasticities are respectively 1 and $\frac{1}{\theta}$ for consumption and leisure. γ_j denotes the relative preference for leisure versus consumption. This preference parameter is age-specific but not ability dependent. Eq. (4) and (5) will be the two driving forces for household optimization. Of course, agents are limited in their time and resources. These constraints are covered in the next section.

[9] For completion: $sr_8^t = 0$, and thus $\pi_8^t = 0$, which corresponds to the premise of absolute death at age 100. For a visual summary of the life-cycle, including the link between ages and survival, I refer to figure 3.1.

3.3.2 Time and budget constraints

Time Constraints

Agents have an endowment of time: in any period of time agents own one unit of time. They can split this unit of time in different fractions. What agents can do with their time depends on their age and their ability. The young (1st period of life) and medium or highly able individuals can spend time on education (e_S^t), work ($n_{j,s}^t$) or leisure ($l_{j,s}^t$). Due to a lack of productivity in schooling, low ability individuals don't spend any time on education. After the first period, but during the active period of life (i.e. the periods 2-5) agents only chose between work and leisure. After choosing when to retire in the 5th period (R_s^t) agents only spend time on leisure (periods 6-8).

Model-wise the following equations represent the time constraints in which agents operate. The equations for individuals born at time t and of ability s are reported below:

1st period

$$\textit{For medium and high ability individuals}$$

$$For\ j\ =\ 1\ (ages\ 20-29) \qquad : \qquad l_{1,s}^t = 1 - n_{1,s}^t - e_S^t \tag{6.a}$$

$$\textit{For low ability individuals}$$

$$For\ j = 1\ (ages\ 20-29) \qquad : \qquad l_{1,L}^t = 1 - n_{1,L}^t \tag{6.b}$$

2nd period - 4th period

$$\textit{Mutual amongst all ability households}$$

$$For\ j\ =\ 2-4\ (ages\ 30-59) \qquad : \qquad l_{j,s}^t = 1 - n_{j,s}^t \tag{7}$$

5th period

Mutual amongst all ability households

$For\ \ j = 5\ (ages\ 60 - 69):$

$$l^t_{5,s} = \mathrm{Z}\left(\mu\left(R^t_s(1-\tilde{\mathrm{n}}^t_{5,s})\right)^{1-\frac{1}{\zeta}} + (1-\mu)(1-R^t_s)^{1-\frac{1}{\zeta}}\right)^{\frac{\zeta}{\zeta-1}} \qquad (8)$$

6th to 8th period

Mutual amongst all ability households

$For\ \ j = 6\ to\ 8\ (ages\ 70 - 99):$

$$l^t_{j,s} = 1$$

To model the choice for the retirement age (R^t_s), a CES leisure function for the 5th period of life is introduced, based on the work of Buyse et al (2013, 2017). The reason for introducing a leisure function is twofold: First, leisure before and after retirement might be of a different kind and thus yield different utilities. As a result agents will prefer a balanced combination of both types of leisure. Second, this enables us to give the same meaning to γ_5 and make it comparable to the other relative values of leisure (γ_1 ,γ_2 ,γ_3 and γ_4). These γ_j are the preference parameters for leisure, which are age dependent but not ability dependent. In the equations (6) to (8), this can be recognized by the age subscript j and the absence of the ability subscript s.

An alternative way of modelling the retirement choice is to consider the utility function instead of the time constraint. An example of this approach can be found in the work of de La Croix et al (2012)[10] and Dedry et al (2017)[11].

[10] In de la Croix et al (2012) agents receive utility by retiring early. This additional utility is represented by a concave function, showing the diminishing returns to early retirement.
[11] In Dedry et al (2017) working in the second period creates disutility, but at the same time this disutility is an negative function of longevity.

Below follows an explanation of the different symbols used in the time constraints eq. (6) to eq.(8):

Z is the normalization parameter to ensure that $0 < l_{5,s}^{t} < 1$.

μ is the relative pleasure given to pre-retirement leisure, which makes $(1-\mu)$ the relative weight given to leisure during retirement. They are the share parameters.

R_s^t is the fraction of time still active on the labour market. Making $(1-R_s^t)$ the fraction of time in retirement.

$\tilde{n}_{5,s}^{t}$ is hours worked as a fraction of the time when still active on the labour market. In this regard, the following equation holds: $n_{5,s}^{t} = R_s^t \tilde{n}_{5,s}^{t}$. Which states that total fraction of time worked in the 5th period, equals the fraction of time on the labour market (R_s^t) times the amount of hours worked while being on the labour market ($\tilde{n}_{5,s}^{t}$).

Lastly ζ is the elasticity of substitution between both types of leisure.

Fig 3.1 Life-cycle and time constraints

Age	20–30	30–40	40–50	50–60	60–70 (R_s^t)	70–80	80–90	90–100
	sr_1^t	sr_2^t	sr_3^t	sr_4^t	sr_5^t	sr_6^t	sr_7^t	sr_8^t

Active life | Retirement

Period	t	t+1	t+2	t+3	t+4		t+5	t+6	t+7
Work	$n_{1,s}^t$	$n_{2,s}^t$	$n_{3,s}^t$	$n_{4,s}^t$	$\tilde{n}_{5,s}^t$	0	0	0	0
Study	$e_{1,s}^t$	0	0	0	0	0	0	0	0
Leisure	$1 - n_{1,s}^t - e_{1,s}^t$	$1 - n_{2,s}^t$	$1 - n_{3,s}^t$	$1 - n_{4,s}^t$	$1 - \tilde{n}_{5,s}^t$	1	1	1	1

For low ability individuals $e_{1,L}^t = 0$
R_s^t is the fraction of time that agents are still active on the labour market, this is endogenously decided by agents.
Superscript t denotes the generation of a cohort, referring to the period in which they were born.
Subscripts s denote ability type and the other subscript denotes the age this cohort has.

Budget Constraints

Individuals are also constrained in their consumption possibilities. They consume and save only what they earn through previous savings and through current labour income. The budget constraints are quite similar across age cohorts, with exception of the retired cohorts and workers in their last period of active life. The equations for the budget constraints of individuals born at time t and of ability type s are reported below:

Period 1 – 4 (ages 20 – 59):

$$(1+\tau_c)c_{j,s}^t + a_{j,s}^t = \left(1+r_{t+j-1}\right)\left(a_{j-1,s}^t + ab_{t+j-1}\right) + w_{t+j-1}^s \varepsilon_j h_{j,s}^t n_{j,s}^t (1-\tau_w - pf_1) + z_{t+j-1} \quad (9)$$

The Left-Hand Side (LHS) shows the consumption and the accrued non-human wealth:

$c_{j,s}^t$ represents the consumption of the agent at age j, on this consumption a consumption tax is levied (τ_c). What is left from the disposable income goes to $a_{j,s}^t$, which stands for non-human wealth accumulated at the end of the j-th period of life.

The Right-Hand Side (RHS) displays the total disposable income, consisting of:

$a_{j-1,s}^t$ represents the non-human wealth accumulated from the previous period and ab_{t+j-1}. are the accidental bequests (see further) , agents receive an interest payment (r_{t+j-1}) on the sum of $a_{j-1,s}^t$ and ab_{t+j-1}.

$w_{t+j-1}^s \varepsilon_j h_{j,s}^t n_{j,s}^t$ denotes the before tax wage income of the agent at time j, which depends on the wage rate w_{t+j-1}^s, on the age productivity ε_j, on the human capital stock $h_{j,s}^t$ and on hours worked $n_{j,s}^t$ during the period j.

On this labour income the government levies a tax and the firm withholds a sum:

τ_w denotes the labour income tax rate, which also includes the employees' contribution rate (cr_1) to the first pension pillar.

pf_1 represents the employees' contribution rate to the second pillar pension fund.

Lastly, z_{t+j-1} stands for the lump-sum transfer that the governments bestows upon all households.

An important note is that individuals don't leave intentional bequests and as such agents don't receive an inheritance, leaving $a_0 = 0$.

Whilst individuals do not intentionally leave bequests for the next generation, they do receive bequests in the form of accidental bequests (ab_{t+j-1}^{t}). During each period a fraction of each generation dies. The fraction $\left(1 - sr_j^t\right) * N_j^t$ is the part of generation t who died in their j-th period of life and is a direct consequence of eq. (2). This same principle applies to all ages and all generations. These deceased individuals have accrued non-human wealth and credit (see second pension fund) during their lifetime, but at death this wealth is redistributed equally to all the remaining living cohorts (ab_k). Just like regular non-human wealth, these accidental bequests also yield interest, which gives additional income in the following period.

$$ab_k = \frac{1}{N_k}\left[\sum_s \sum_{j=1}^{5}\left(1 - sr_j^{k-j}\right)\left(a_{j,s}^{k-j} + spp_{j,s}^{k-j}\right) N_{j,s}^{k-j} + \sum_s \sum_{j=6}^{8}\left(1 - sr_j^{k-j}\right) a_{j,s}^{k-j} N_{j,s}^{k-j}\right]$$

Where spp_s^{k-j} refers to the second pension fund credit that individuals of generation k-j have accrued before reaching period k.

Period 5 (ages 60-69) :

$$\begin{aligned}(1+\tau_c)c_{j,s}^t + a_{j,s}^t &= \left(1 + r_{t+j-1}\right)\left(a_{j-1,s}^t + ab_{t+j-1}\right) + (1 - R_s^t) ppt_s^t (1 + BMR)^{\kappa(R_s^t - \rho)} \\ &+ spp_s^t + R_s^t\left[w_{t+j-1}^s \varepsilon_j h_{j,s}^t \tilde{n}_{5,s}^t (1 - \tau_w - pf1)\right] + z_{t+j-1}\end{aligned} \quad (10)$$

With $0 < R_s^t < 1$

$0 \leq BMR$ and $\kappa > 0$

Similarly as before, the LHS consists out of consumption and the accrued non-human wealth in the j-th period of life.

The RHS is similar for the interest received on previous accrued non-human wealth and for the accidental bequests. Work related income is different however, there are two parts depending on whether a worker has retired.

This can be seen by the symbol R_s^t , this denotes the chosen fraction of time that a worker is still active on the labour market. Such that $(1 - R_s^t)$ becomes the fraction of time that the worker has retired. Whilst the worker is still active on the labour market, he receives an income quite similar to eq. (9). For the part that he is retired he receives a first and a second pension pillar benefit.

ppt_s^t is the first pillar pension benefit that an individual receives. Added to that, the government rewards late retirement by paying a bonus $\mathrm{ppt}_s^t * BMR$. The Bonus Malus Rate (BMR) can be any positive number and acts both as a bonus and as a malus rate. The fraction of time related to the official retirement age is denoted by ρ. If agents retire before the official retirement age, they get a reduction in pension payments :

If the $R_s^t > \rho$, then $(1 + BMR)^{\kappa(R_s^t - \rho)}$ becomes larger than 1 and acts as a bonus.
If the $R_s^t < \rho$, then $(1 + BMR)^{\kappa(R_s^t - \rho)}$ becomes smaller than 1 and acts as a malus.

κ serves as a normalization parameter, such that the BMR can be seen as a yearly rate. This value will change depending on the years that represent one period in a model. In this model one period equals 10 years and as such $\kappa = 10$. Subsequent, $\kappa * R_s^t$ becomes the amount of years after 60 that an individual is still active and $\kappa * \rho$ represents the official retirement age, expressed in years over 60. It should be noted that ρ serves as a cut-off point in the bonus-malus. When $R_s^t = \rho$ no bonus or malus is awarded.

spp_s^t denotes the second pillar pension benefit, which is a fully funded one-time payment upon retirement (see following section).

The last part is income through work while still on the labour market $R_s^t\left[w_{t+j-1}^s \varepsilon_j h_{j,s}^t \tilde{n}_{5,s}^t (1 - \tau_w - pf1)\ \right]$, which follows the same principles as in previous periods of life. Lastly, z_{t+j-1} is the lump-sum transfer.

Periods 6 – 8 (ages 70 – 99) :

$$(1+\tau_c)c_{j,s}^t + a_{j,s}^t = \left(1+r_{t+j-1}\right)\left(a_{j-1,s}^t + ab_{t+j-1}\right) + ppt_s^t(1+BMR)^{\kappa(R_s^t-\rho)} + z_{t+j-1} \quad (11)$$

With $a_8^t = 0$

In later periods of retirement equation (10) simplifies to equation (11): there is no active work life and no second pillar pension fund. The rest is equal with the previous budget constraint.

Mirroring previous constraints : the LHS consists out of consumption and accrued non-human wealth during the j-th period of life.

The RHS consists of interest received on accumulated non-human wealth of the previous period and the accidental bequest. Next part are the pension benefits multiplied with the bonus or malus, depending on the choice agents made in the 5th period.
Lastly, z_{t+j-1} is the lump-sum transfer that all individuals receive.

First and Second Pillar pension

Both pension pillars are related to previous earnings of individuals. What follows is how those earning are linked to the pension benefits. For the first pillar:

The first pension pillar is a public PAYG pension scheme of the defined benefits type. Contributions from the current active population funds the payments of the current retired population. When pension payments and contributions don't match, a pension deficit or surplus is created. This pension deficit/surpus is taken into the general budget of the government (cfr. 3.5 Government).

Below is shown how an individual(born at time t and of ability s)'s previous earnings relate to his pension benefits:

$$ppt_{5,s}^{t} = rr_s\left(\sum_{j=1}^{4} p_j w_{t+j-1}^{s}\varepsilon_j h_{j,s}^{t} n_{j,s}^{t}(1-\tau_w) + p_5 w_{t+4}^{s}\varepsilon_5 h_{5,s}^{t} R_s^t \tilde{n}_{5,s}^{t}(1-\tau_w)\right) \qquad (12.a)$$

Pension benefits are a weighted average of the previous earnings during active life. Because an individual works for 5 periods the pension benefit will be the sum of the accrued benefits of these periods. The accrual rate within each period consists out of the net replacement rate and the pension weight attached to that period of work.

rr_S is the net replacement rate for pensions. Pensions are progressively modelled, which means that $rr_L > rr_M > rr_H$.

The different p_j denote the pension weights attached to earnings during the j-th period of life for the calculation of the pension benefit. Being weights, this follows: $0 < p_j < 1$ and $\sum p_j = 1$.

Because of the retirement choice the accrued benefits in the last period of active life (ages 60-69) need to be corrected for the retirement age. Only for the period still active on the labour market (R_s^t) does an agent accrue benefits. From the time constraint eq. (8) follows this relationship $n_{5,s}^t = R_s^t * \tilde{n}_{5,s}^t$ which can be used to simplify eq. (12.a) to eq. (12.b):

$$ppt_{5,s}^t = rr_s\left(\sum_{j=1}^{5} p_j w_{t+j-1}^s \varepsilon_j h_{j,s}^t n_{j,s}^t (1-\tau_w)\right) \tag{12.b}$$

The initial pension payment $ppt_{5,s}^t$ is not revaluated to adjust for changing living standards, as a result $ppt_{j,s}^t = ppt_{5,s}^t \quad for\ j = 6,7,8.$

The second pension pillar is a fully funded pension scheme and consists out of payments by both workers and firms. Those payments build up credit. Credit increases over time due to the additional payments to the pension fund and due to interest accrued on previous credit.

$$spp_s^t = \sum_{j=1}^{4} credit_{j,s}^t \prod_{i=j}^{4}(1+r_{t+i}) + credit_{5,s}^t \qquad \forall s \tag{13}$$

$$credit_{j,s}^t = (pf_1 + pf_2) w_{t+j-1}^s h_j^t \varepsilon_j n_{j,s}^t$$

The level of income and benefits shown in eq.(9) to eq.(13) depend on how productive individuals are when working. The basis of the productivity lies in the educational choices that individual make in their first period of life. The next section examines how individuals invest in education and how this translates into accumulated human capital.

3.3.3 Human Capital

Agents enter the model with a predetermined stock of human capital. Depending on their learning capacities they choose to study a fraction of their available time. If they spend time on education, this translates into an increase of their human capital stock in the next period. An important restriction is that agents are only allowed to study during their first period of life (age 20 to 29).

An agent's initial human capital stock depends on his innate ability:

$$h_{1,s}^{t} = \theta_s h_0 \qquad \forall s \tag{14}$$

Where $\theta_H = 1$

$0 < \theta_L < \theta_M < 1$

$h_0 > 0$

θ_s shows how much initial human capital stock a low and a medium ability agent have relative to a high ability individual. High ability individuals have h_0 initial stock of human capital and out of eq. (14) follows that: $0 < h_{1,L} < h_{1,M} < h_{1,H}$. Which states that high ability individuals have the highest initial stock of human capital, followed by medium and low ability individuals respectively.

When agents spend time studying, their human capital increases as follows:

$$h_{j+1,s}^{t} = h_{j,s}^{t}\left(1 + \phi_s\left(e_{j,s}^{t}\right)^{\sigma}\right) \qquad for\ s = \{M, H\} \tag{15}$$

With $\sigma > 0$

$\phi_s > 0$

One has to note that eq. (15) only holds for medium and high ability individuals. Low ability individuals do not spend time in education, and thus do not increase their initial human capital stock. This is optimal behaviour as low ability agents lack productivity in schooling. The ability-specific efficiency parameter that reflects this productivity of schooling is denoted by ϕ_s[12]. The model assumes that high ability agents are more efficient in education than medium ability agents, such that: $\phi_M < \phi_H$. Lastly, σ denotes the elasticity of human capital growth with respect to time spent in

[12] Alternatively eq. (15) could include low ability agents, conditional on $\phi_L = 0$.

education.

In further periods of life the human capital stock remains at the level of the second period. The reasoning behind this is that learning-by-doing and human capital depreciation offset each other. This applies to all ability types, the low ability agents however do not study and as a result their human capital stock is always at the level of their initial stock of capital stock.

$$h_{2+i,s}^{t} = h_{2,s}^{t} \qquad for\ s = \{M, H\}\ and\ i = \{1,2,3,4\} \tag{16.a}$$

$$h_{j,L}^{t} = h_{1,L}^{t} \qquad j = \{2,3,4,5\} \tag{16.b}$$

3.3.4 Household optimization

Households optimize their lifetime utility by choosing the optimal amount of consumption and labour supply as to maximize equation (4) whilst taking into account their instantaneous utility function - equation (5) - and their budget and time constraints - equation (6) to (13) -. Agents also choose the optimal retirement age based on its impact through the budget constraints eq.(9), (10), (11), (12.a) and time constraints eq.(8). High and medium able agents also optimally choose education, taking the human capital production eq.(14), (15) and (16.a) and the budget and time constraints - equations (7) to (13) - into account. For further details on the first-order conditions of households, I refer to Appendix A.

3.4 Firms

Firms are acting on perfectly competitive in- and output markets. Firms are identical, which combined with the free entry condition makes them of equal size. We can thus resort to the case of one firm and take that result as being representative for all firms. To model production, a Cobb-Douglas production function is introduced. The production function is expressed in eq.(17):

$$Y_t = K_t^{\alpha}(A_t H_t)^{1-\alpha} \tag{17}$$

With $0 < \alpha < 1$

Where Y_t is production, K_t is the stock of physical capital, A_t is the level of labour augmenting technological progress and H_t is the effective labour being used. Technology grows at an exogenous rate $g_{a,t}$:

$$A_t = g_{a,t} A_{t-1} \tag{18}$$

With $g_{a,t} > 1$

Effective Labour H_t is itself a CES-function, which is represented as follows:

$$H_t = \left(\eta_L H_{L,t}^{1-\frac{1}{\lambda}} + \eta_M H_{M,t}^{1-\frac{1}{\lambda}} + \eta_H H_{H,t}^{1-\frac{1}{\lambda}}\right)^{\frac{\lambda}{\lambda-1}} \tag{19}$$

Where the different η_s represent the share parameters of the different ability types.
Being share parameters, they add up to 1: $\eta_L + \eta_M + \eta_H = 1$.
Being a CES-function, λ operates as the elasticity of substitution between different ability types of workers in effective labour.

$H_{s,t}$ stands for the total effective labour of the ability type s at time t and is calculated as follows:

$$H_{s,t} = \sum_{j=1}^{5} N_{j,s}^{t-j+1} \left(n_{j,s}^{t-j+1} \varepsilon_j h_{j,s}^{t-j+1} \right) \quad \forall s \tag{20}$$

In which the sum goes over all age groups and the product is within one age group. With $N_{j,s}^{t}$ represents the size of the cohort born at time t with ability s and age j.

$n_{j,s}^{t}$ is hours worked by an individual of generation t with ability s at age j.

ε_j is the age productivity parameter linked to age j.

Lastly, $h_{j,s}^{t}$ stands for the human capital stock of an individual of generation t with ability s at age j.

3.4.1 Firm optimization

Firms face the same profit maximalization problem in every period. Instead of solving the dynamic problem, firms are solving the static problem at each period in time. As such the optimization problem simplifies immensely.

Firms will attract capital and effective labour until their marginal costs equalize their marginal benefits, which results in the following FOC's for firms:

$$(1-\alpha) A_t \left(\frac{K_t}{A_t H_t} \right)^{\alpha} \eta_s \left(\frac{H_t}{H_{s,t}} \right)^{\frac{1}{\lambda}} = w_t^s (1 + \tau_p + pf_2) \tag{21}$$

Because of the assumption of a competitive labour market, the total demand and supply of effective labour will equalize. The wage cost per effective unit of labour will be equal to the marginal labour productivity. This FOC is shown in eq. (21): with the marginal productivity per effective labour of ability type s on the LHS and the wage cost per effective unit of labour of ability type s on the RHS. τ_p is the employer social contribution rate, which includes the employers' contribution rate (cr_2) to the first pillar pension system and pf_2 denotes the employers' contribution rate to the second pillar pension fund. The firm will take these costs (τ_p and pf_2) into account when deciding to hire workers.

Likewise, competition will ensure that the supply and the demand for capital equalize. The households willingness to save and the firms desire to invest will determine the interest rate at which the capital market clears :

$$\left[\alpha\left(\frac{A_t H_t}{K_t}\right)^{1-\alpha} - \delta\right](1-\tau_k) = r_t \tag{22}$$

With $0 < \delta < 1$

Firms will install capital up to the point where the after-tax $(1-\tau_k)$ marginal product of capital, net of depreciation (δ), equals the interest rate (r_t). If the interest rate is higher than the marginal productivity, the demand for capital will decrease in order to ensure eq. (22). Reversely, if the interest rate is lower than the marginal productivity, firms will increase their demand until eq.(22) holds. The supply of capital comes from the national stock of wealth. A part of this stock is reserved to finance the governments outstanding debt (see 3.6 General equilibrium).

3.5 Government

The government has a general budget and a pension budget. Eq. (23) reflects the former:

$$B_{t+1} - B_t = B_t r_t + G + GPP_t + Z_t - T_{nt} - T_{pt} - T_{kt} - T_{ct} \tag{23}$$

On the LHS we find the change in government debt. On the RHS, the first 4 terms are the expenditures and the last 4 terms represent the revenues of the central government.

<u>The expenditures are</u>:

$B_t r_t$ which stands for the interest payments on outstanding debt.

G_t denotes the level of government consumption. The government consumption is assumed to be a fraction of total output. Such that: $G_t = g_t\, Y_t$. An additional assumption is that the government consumption is non-productive.

GPP_t is the first pillar pension deficit that is covered by the general government (see following section on the pension budget).

Lasrly, Z_t are the total lump sum transfers to all households:

$$Z_t = z_t \sum_{j=1}^{8} N_j^{t-j+1} \tag{24}$$

<u>The revenues are:</u>

T_{nt} denotes the total taxation on labour income and is calculated in the following way:

$$T_{nt} = \sum_{s} \sum_{j=1}^{5} N_{j,s}^{t-j+1} w_t^s h_{j,s}^{t-j+1} n_{j,s}^{t-j+1} \varepsilon_j (\tau_w - cr_1) \tag{25}$$

T_{pt} represents the total social contributions of employers:

$$T_{pt} = \sum_{s} \sum_{j=1}^{5} N_{j,s}^{t-j+1} w_t^s h_{j,s}^{t-j+1} n_{j,s}^{t-j+1} \varepsilon_j (\tau_p - cr_2) \tag{26}$$

T_{kt} symbolises the total taxation on capital:

$$T_{kt} = \tau_k K_t \left[\alpha \frac{Y_t}{K_t} - \delta \right] \tag{27}$$

T_{ct} expresses the total taxation on consumption

$$T_{ct} = \tau_{ct} \sum_{s} \sum_{j=1}^{8} N_{j,s}^{t-j+1} \, c_{j,s}^{t-j+1} \tag{28}$$

<u>The pension budget</u>

The pension budget consists of contributions of the active population and the pension payments to the retired population. If total contributions don't match total pension payments a deficit or surplus is created.

$$GPP_t = \sum_{s} \sum_{j=5}^{8} N_{j,s}^{t-j+1} ppt_{j,s}^{t-j+1} - cr \left(\sum_{s} \sum_{j=1}^{5} N_{j,s}^{t-j+1} \left(n_{j,s}^{t-j+1} \varepsilon_j h_{j,s}^{t-j+1} \right) \right) \tag{29}$$

With $cr = cr_1 + cr_2$

On the LHS we find the pension budget deficit that the government covers within its general budget. On the RHS we find two parts in double summation. The first part are the pension payments paid to the current old (age cohorts 5 to 8) multiplied by the size of that cohort. The second part are the contribution paid by the working population. With cr encompassing both the employers' (cr_2) and the employees' (cr_1) contribution rates to the first pension pillar.

3.6 General equilibrium

A closed economy implies that capital cannot enter or exit the economy. Eq. (21) and (22) reflect the equilibrium on the factor markets for labour and capital. Equilibrium on the goods market takes the following form:

$$Y_t = C_t + I_t + G_t \tag{30}$$

This equation states that output equals the sum of consumption, investments and government spending. With:

$$I_t = \Delta K_{t+1} + \delta K_t \tag{31}$$

Stating that investments are used for accumulating more capital in the next period (1st term RHS) and to compensate for depreciation of the current capital stock (2nd term RHS).

Lastly:

$$\Omega_t + SPP_t = K_t + B_t \tag{32}$$

Where Ω_t stands for the total private wealth held by households and SPP_t stands for the accumulated wealth from the second-pillar pension fund. These assets can be allocated either to physical capital (K_t) or government debt (B_t).

With:

$$\Omega_t = \sum_s \sum_{j=1}^{8} N_s^{t-j+1} a_s^{t-j+1} \tag{33}$$

Eq. (33) states that the total private wealth of households is the summation over ability type and age cohorts of the individual accrued wealth of those agents at the end of period t.

4 Parameters and Calibration

In the previous chapter the model was built. In this chapter values are attributed to the parameters within the model. There are two sets of parameters: (i) those rising up from literature and (ii) those determined by calibration.

4.1 Preference and technology parameters

In the following section a distinction is made between parameter values that rise out of literature and parameters that are determined through calibration.

4.1.1 Parameters from literature

Using the current literature the value of different parameters can be set. The intertemporal elasticity of substitution in leisure is set to ½, such that its inverse $\theta = 2$. This value is taken from Devriendt and Heylen (forthcoming) and is the halfway point of the range set by Rogerson (2007). In micro studies one normally finds much higher values of θ. But as Rogerson and Wallenius (2009) show, micro elasticities might not hold for macro studies.

Regarding physical capital, the depreciation rate δ is set at 0.383. This corresponds to a yearly rate of 4.7% and is taken from Penn World Table 8.1 (Feenstra, 2007). This rate equals the average depreciation rate for Japan. Following Devriendt and Heylen (forthcoming) the share of capital in production α is set equal to 0.375.

In order to determine the age-specific productivity parameter ε_j, this dissertation uses the formula as stated in Miles (1999) and Cournède and Gonand (2006) to achieve the hump-shaped modelling of the age-specific productivity parameter. Their formula is: $\varepsilon_j = exp(0.05age - 0.0006age^2)$. Where age is expressed as the age in years and not in terms of periods of life. For each period of life the average productivity parameter is calculated based on the actual ages represented in that period. For the first period of life e.g. the productivity of ages 20 to 29 are calculated and the average of those values is used as the productivity parameter of young workers (ε_1).

To avoid the overestimation of the returns to education, this dissertation follows the approach from Devriendt and Heylen (forthcoming) and Buyse et al (2013, 2017). These authors set $\sigma = 0.3$, which is low but within the range set forward by Bouzahzah et al. (2002) and Docquier and Paddison (2003). However, this value is much lower than what is used by Lucas (1990).

Unfortunately there is less guidance for the determination of the initial human capital stock. To pin the relative initial human capital stock of an ability type this dissertation follows Buyse et al.(2017). These authors use PISA science scores. Whilst they have used Belgian data, the same method has been followed using Japanese data. Students at the 17th percentile are considered representative for low ability individuals, those at the 50th percentile are representative for medium ability individuals and those at the 83rd percentile are considered representative for the high ability individuals. Using the relative PISA scores one finds $\theta_L = 0.7158$ and $\theta_M = 0.8579$.

For the elasticity of substitution between ability types in effective labour, this dissertation uses Devriendt and Heylen (forthcoming) and Buyse et al (2017)'s value and set $\lambda = 1.5$. This value is fully within the range reported by the empirical labour literature (Caselli & Coleman, 2006)

Regarding the CES leisure function, the following values are taken from Buyse et al.(2017) : $Z = 2$ and $\mu = 0.5$. Since there is no indication that one type of leisure is preferred to the other type, equal weight is given to μ and $(1 - \mu)$. In this case the normalization parameter Z needs to be set to 2.

As a reminder, the share parameters v_L, v_M and v_H are set equal to $1/3$. This assumption is also made by Devriendt and Heylen (fortcoming) and Buyse et al. (2017). Whilst this must not be the case in real-life, the assumption is not so farfetched. It is known that I.Q. is normally distributed. If one assumes that I.Q. is a sign of innate ability, than the symmetry of the distribution allows us to easily divide the population in 3 equal-sized groups of different ability.

Initially the rate of time preference was taken from Barro (1990) and β set to 0.817. Because one period equals 10 years, this corresponds to a yearly rate of 2%. Unfortunately this did not yield a good fit with regards to the capital output ratio. Since the model has survival rates, it made sense to also calibrate the time preference. This is logical as it is not the pure time preference, but the effective time preference that will determine the saving behaviour of individuals.

What remains are 12 parameters that need to be determined. These are determined through calibration, which is explained in the following section.

4.1.2 Parameters through Calibration

The remaining parameters are the 5 taste parameters for leisure ($\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5$), 3 share parameters for effective labour (η_L, η_M, η_H), 2 efficiency parameters regarding education (ϕ_M, ϕ_H) , the elasticity of substitution in leisure (ζ) and the time preference (β). To calibrate these parameters one can follow the procedure set out in Ludwig et al. (2011):

1. In a first step one makes an initial guess for the parameters by calibrating for a steady state. The calibration period in this dissertation is 1995-2004 and demographics are frozen at the level of 1995-2004 to impose a stationary population. The parameters are chosen to ensure that the model matches the key data obtained in the calibration period. The link between parameters and variables are explained in the next section. The key variables used for calibration are hours worked by age, educational attainment, the number of hours worked per capita, the wage differential between ability groups and the capital-output ratio.

2. The parameters from the initial guess (step 1) are used to create an artificial initial steady state. The start point is 1945. In this step demographics are still imposed to be stationary.

3. Demographics, policy variables and technological progress are introduced as exogenous forces that drive the model. These exogenous variables push the model through a transition path towards a new steady state.

4. The transition path goes through the calibration period (1995-2004). One then compares the simulated averages with the real averages(=targets) of the key variables within the calibration period. The ratio of the simulated variables and the targets are then used as overshoot ratios.

5. These overshoot ratios are used to alter the parameter values. The parameter values are then divided by the overshoot ratios to obtain a new guess of these parameter.

6. One repeats step 1 to 5 until the differences between the real and simulated moments in 1995-2004 are minimized.

Whilst all of these parameters are determined simultaneous, some parameters are closely linked to the targets used for calibration.

It is clear that the preferences for leisure (γ_1 , γ_2 , γ_3 , γ_4 , γ_5) are closely linked to the hours worked (n_1 , n_2 , n_3 , n_4 , n_5). Likewise, the efficiency parameters in education (ϕ_M , ϕ_H) are closely linked to the participation rates in higher education (e_M , e_H). The retirement choice (R) is also linked to the elasticity of substitution of leisure in the fifth period of life(ζ).

The share parameters η_L and η_M are determined to match observed pre-tax earnings of workers with different abilities. Data on relative earnings are obtained from the OECD, education at glance. Data on wages of workers between 25 and 64 years old are used to determine relative earnings. Earnings of workers without higher secondary education are deemed representative for low able individuals. Earnings of workers with secondary (but no tertiary) schooling are representative for medium able individuals. Lastly earnings of workers with a tertiary degree are representative for high ability individuals. One only has to calibrate for η_L and η_M, as $\eta_H = 1 - \eta_L - \eta_M$.

Finally, and as stated before, $\beta = 0.817$ will be taken as a starting guess. β has been calibrated as such that the capital output ratio reaches the level attained in 1995-2004. The link is clear, the effective time preference is linked to the savings' rate of society. By calibrating β along-side the demographic change, one can expect to attain a closer fit of the capital-output ratio.

Table 4.1: Parametrization of the model

Parameter	Description	Value
Taken from literature		
α	Production share of capital	0.375
λ	Elasticity of substitution between workers of different ability	1.5
θ	Inverse of the intertemporal elasticity to substitute leisure	2
δ	Depreciation rate of physical capital	0.383
σ	Elasticity of human capital with respect to education	0.3
ε_j	Age productivity	$exp(0.05age\ -\ 0.0006age^2)$
$v_L\ ,v_M\ ,v_H$	Share of ability type in total population	$v_L = v_M = v_H = 1/3$
Z	Normalization parameter in the CES leisure function	2
μ	Share parameter in CES leisure function	0.5
$\theta_L\ ,\theta_M$	Relative initial human capital	$\theta_L = 0.7158$ $\theta_M = 0.8579$
Calibrated		
$\gamma_1,\gamma_2,\gamma_3,\gamma_4,\gamma_5$	Preference for leisure	$\gamma_1 = 0.54$ $\gamma_2 = 0.29$ $\gamma_3 = 0.19$ $\gamma_4 = 0.21$ $\gamma_5 = 0.38$
$\eta_L\ ,\eta_M\ ,\eta_H$	Input shares of workers with a certain ability	$\eta_L = 0.27$ $\eta_M = 0.32$ $\eta_H = 0.41$
$\phi_M\ ,\phi_H$	Efficiency parameter of education	$\phi_M = 0.29$ $\phi_H = 1.26$
ζ	Elasticity of substitution in leisure	$\zeta = 0.46$
β	Time preference	$\beta = 0.94$
Targets	Calibration period: 1995-2004	

$n_1 = 0.52$	$n_4 = 0.63$	$w_L/w_H = 0.51$	$e_M = 0.024$
$n_2 = 0.70$	$n_5 = 0.33$	$w_M/w_H = 0.66$	$e_H = 0.192$
$n_3 = 0.71$	$R = 0.60$	$K/Y = 4.08$	

Fiscal and pension policy parameters in the calibration period (averaged over 1995-2004):

$cr_1 = 0.069$	$rr_L = 0.564$	$\rho = 0.5$	$p_1 = p_2 = p_3 = p_4 =$
$cr_2 = 0.069$	$g = 0.357$	$\tau_c = 0.109$	$p_5 = 1/5$
$rr_H = 0.441$	$B/Y = 1.375$	$\tau_w = 0.194$	$\tau_p = 0.134$
$rr\_M = 0.502$	$BMR = 0$	$\tau_k = 0.205$	

5 Validation of the Model

This chapter starts with an outline of the exogenous variables that drive the model. The variables that are covered are the demographic variables, the technological progress and the policy variables.

After displaying the time series of these exogenous variables, this chapter compares key macroeconomic variables of the model - presented in Chapter 3 - to real data. The closeness of fit will determine whether or not the presented model is suitable for simulation and policy evaluation. The focus lies on the following variables: growth in GDP, hours worked, time spent on education and the capital-output ratio. Besides these variables, the fit of demographic change will be evaluated by looking at the dependency ratio.

5.1 Exogenous variables

The following three sections report the time series of the exogenous variables of the model. The three main categories are demographics, technology and the policy variables.

5.1.1 Demographics

The data on demographic change comes from NIPSSR (2017) both series contains historic data and projections for the future. The chosen projection was the medium mortality, medium fertility projection. For the unconditional survival rates (Fig. 5.2) the only rates shown are those for the age groups j = 1 ,3,5 and 7. In Appendix C.1 the interested reader can find the evolution across time of the conditional survival rates of all age groups.

Fig. 5.1 The fertility rate

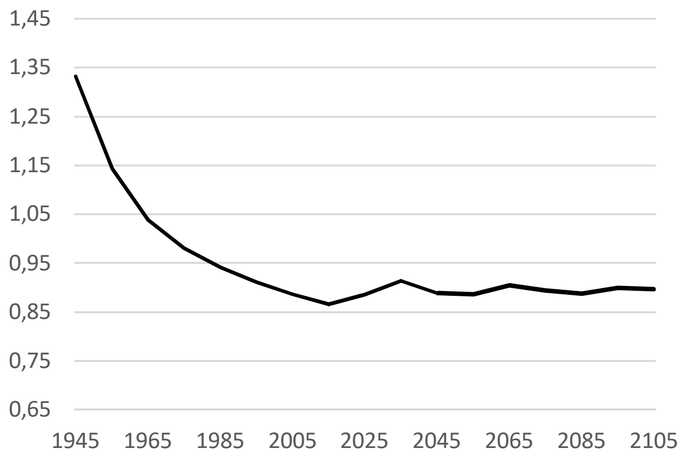
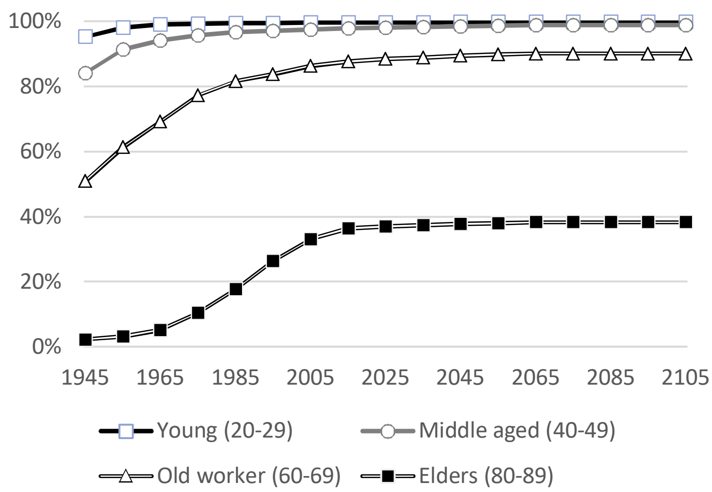
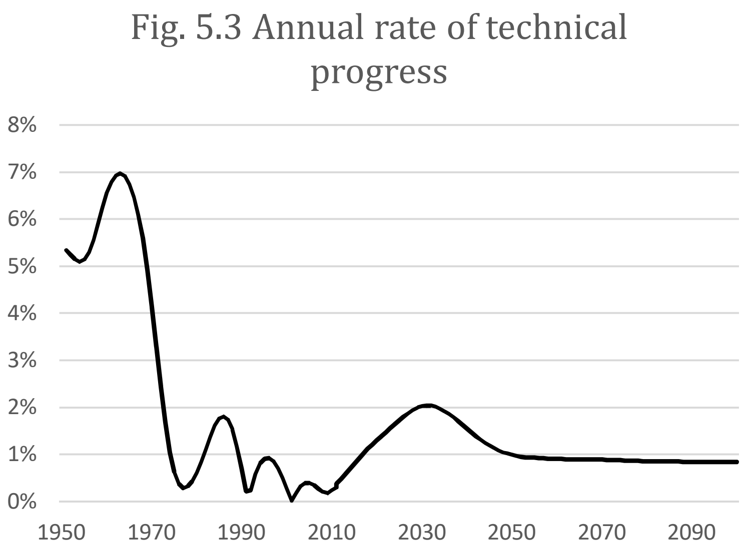


Fig. 5.2 Unconditional survival rates
by age group



5.1.2 Technology

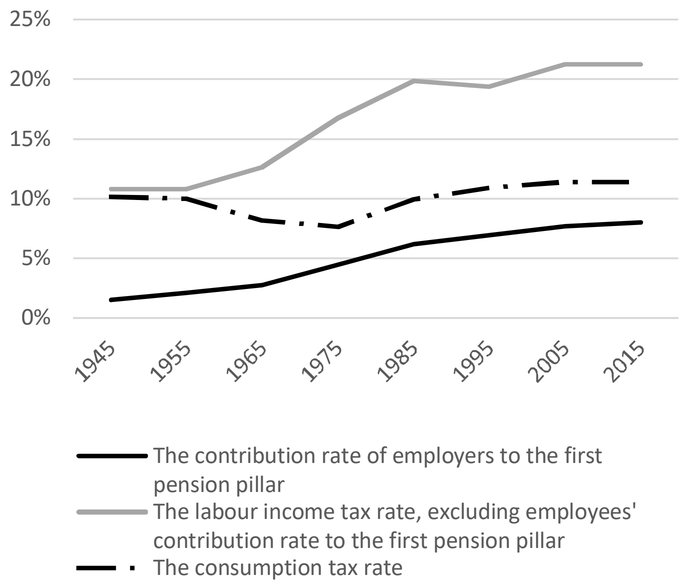
The historic time series on technological progress comes from Penn World Table 8.1 (2015) for which data is used until 2011. From then onwards the historic data and the TFP projections from Cette et al. (2017) are used. These TFP series were adjusted in two ways: (i) a correction was given for a different treatment of hours worked and (ii) the data was smoothed with a HP-filter to obtain a trend rate that excludes cyclical components.



5.1.3 Policy variables

What follows are the time series of the policy variables that are used in the model. The reader finds the construction and the sources of these variables in Appendix B.

Fig. 5.4 Labour and consumption tax rates



Please note that the contribution rate of employers and employees are equal. As such the graph above also plots the time series of the contribution rate for employees.

Fig. 5.5 The marginal corporate tax rate

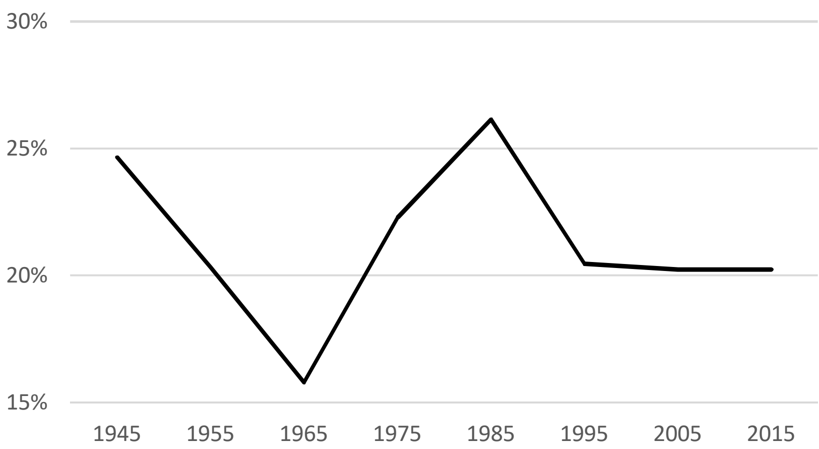


Fig. 5.6 Government consumption
in % of GDP

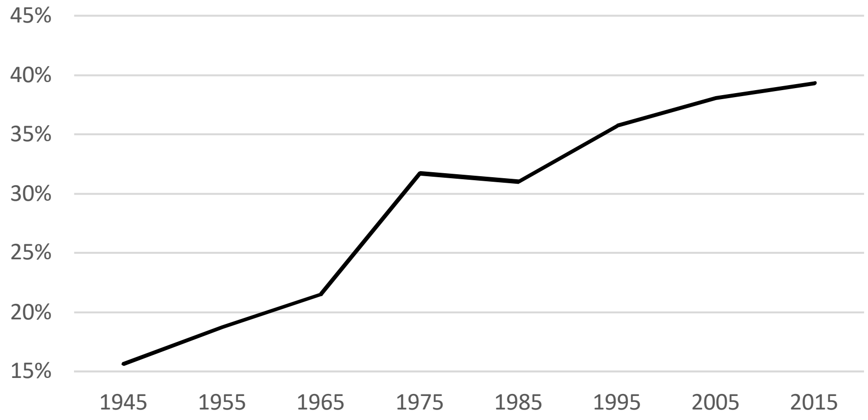


Fig. 5.7 The governments consolidated debt
in % of GDP

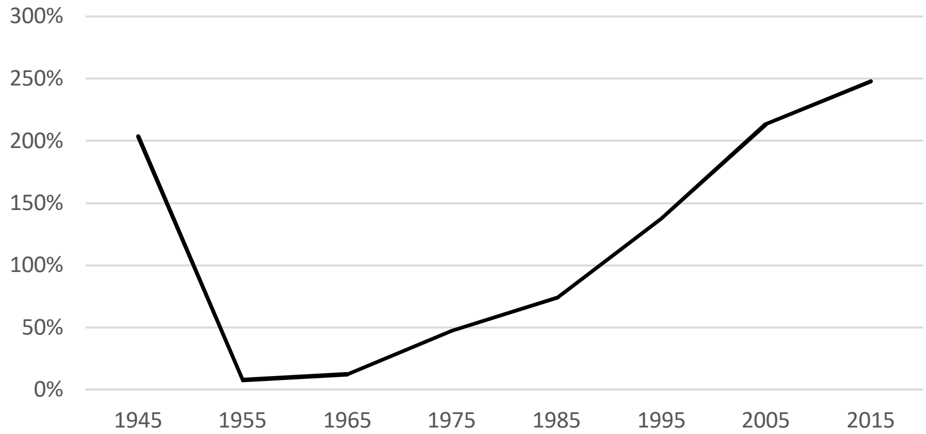
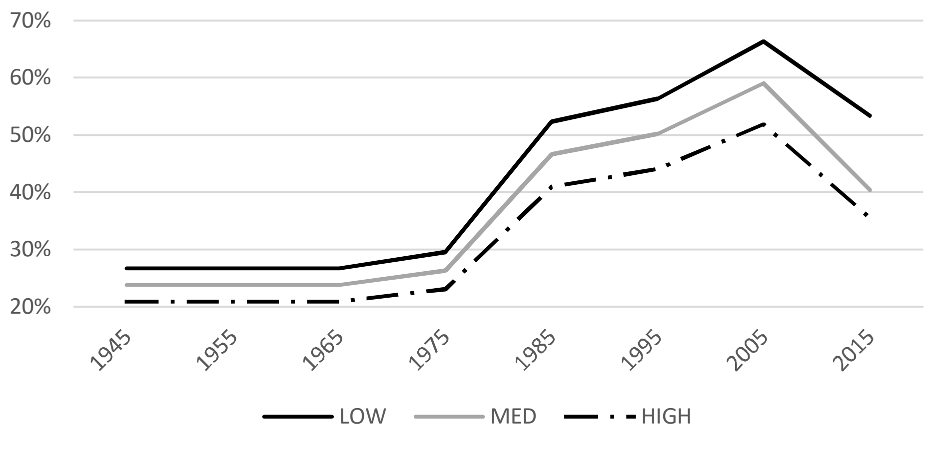


Fig. 5.8 The net replacement rate of public
pensions by ability group

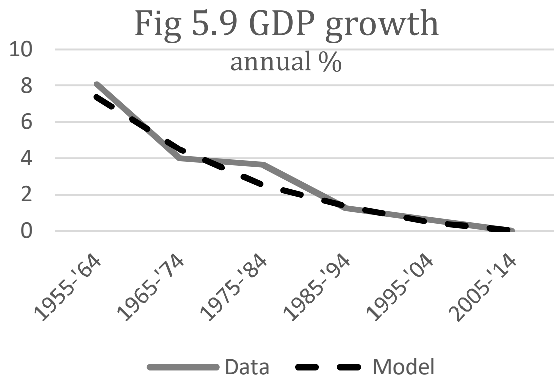


5.2 Goodness of fit

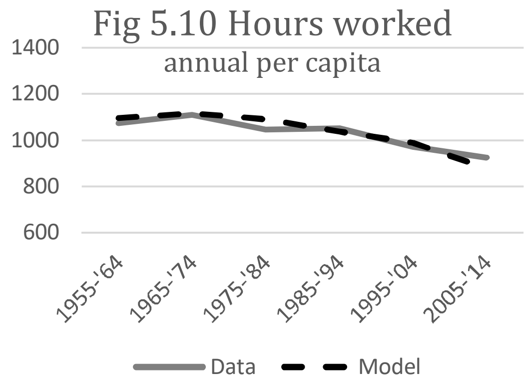
The following subchapter evaluates how well the model's prediction can replicate the past. This backfitting exercise helps to assess whether or not this model is suitable for policy evaluation. When the model can explain key macroeconomic variables it can be realistically assumed that is suited for policy analysis.

5.2.1 GDP and hours worked

Figure 5.9 shows the evolution of the growth rate in aggregate potential GDP over the period 1955-2014. The growth rates shown are averaged annual growth rates as to match the set-up of the model. For comparison sake the 10-year growth rate of the model has been adjusted to yearly growth rates[13]. The predicted values obtained with the model capture the averaged data fairly well, with the exception of the period between 1975 and 1984.



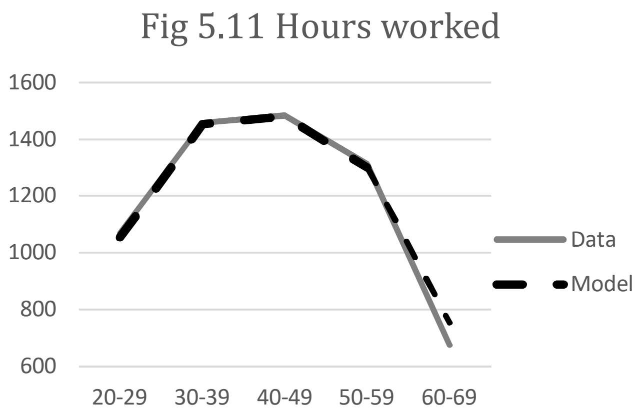
Data source: OECD, Economic outlook 104: potential output volume growth.



Data source: Total economy database (april 2019 ed) : averages over 10 year

Figure 5.10 shows the annual hours worked per capita over the period 1955-2014. To make a sensible comparison between the data and the model, the yearly data was averaged over periods of 10 years, which is consistent with the set-up of the model. The model predicts the fraction of potential time that agents spend on work. The potential time is set to 2080 hours which corresponds to 52 working weeks of 40 hours a week. Whilst not being a perfect match, the observed changes in hours worked follow the data closely. The only exception being the period 1975-1984, for which the model overestimates the hours worked of agents. Likewise, the model slightly overestimates hours worked during the period 2005-2014.

[13] The 10-year growth rate x is transformed as following: (1+x)^1/10 the result of this is (1+y) with y being the yearly growth rate.



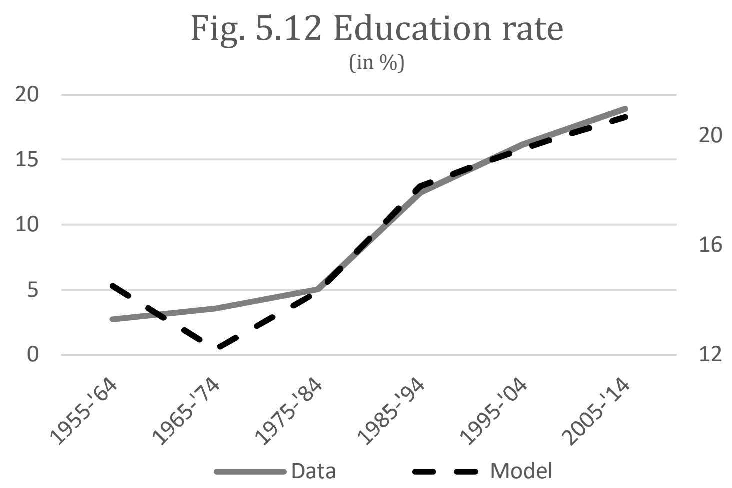
Hours worked by age group during 1995-2004
Data source: Total economy database (april 2019 ed)

Overall the predictions of annual hours worked per capita followed the data closely. The next step is to evaluate the annual hours worked amongst the different age groups. It would have been meaningful to assess as well the hours worked amongst different ability groups. Unfortunately there is no information available on the participation rates linked to educational attainment[14]. This would have complemented the previous back fitting plots. Looking at the different age groupsZ, one can see the tight overlay between the model's prediction and the data.

In the next step education, the capital-output ratio and the old-age dependency ratio will be evaluated between the collected data and the model.

[14] This is the case for other countries, but many data (especially OECD data) is unavailable for Japan.

5.2.2 Education



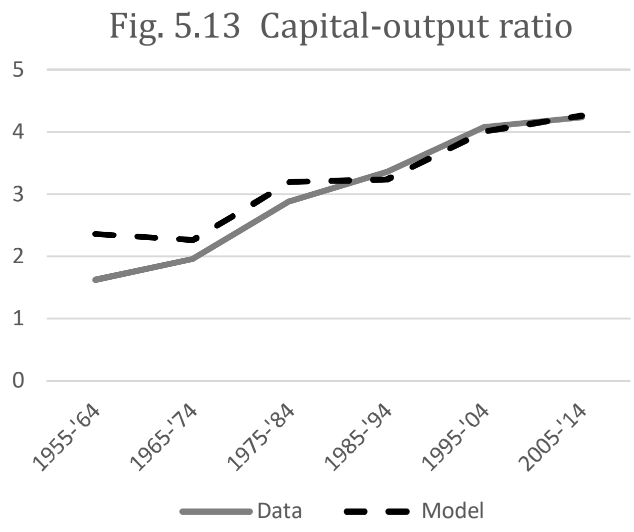
Data (left- scale): completion rate in tertiary education
Model (right-scale): fraction of hours in education (high able)
Data source: Barro and Lee (2013)

S

Looking at education , the reader should note the different scales used. This is to make a comparison possible. The data shows the proportion of the population that has completed tertiary education, whilst the model shows the amount of time high able agents spend on education. Of course these are linked, as such the trend is more important than the level.

Overall we see that the model follows the data well. Especially the increased slope in the mid-1970's followed by a slowing down a decade later.

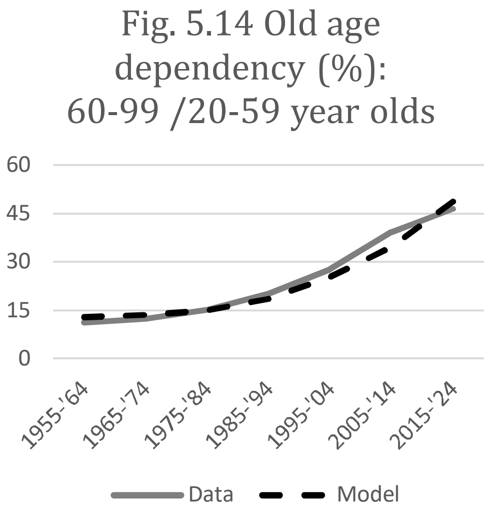
5.2.3 Capital-output ratio



Data source: Penn World Table 8.1, series rkna and rgdpna

Looking at the capital-output ratio, we see that the trend predicted by the model is definitely good. As time progresses, a closer fit between the data and the model emerges. In the first periods the fit between the model and the data is not great, the model over-estimates the capital-output ratio. A possible explanation is that the post-war economy hasn't recovered in the first decades after being hit by a huge shock (i.e. the Second World War). Valdés (2003) states that during WWII the stock of physical capital stock was reduced by a quarter. This is in contrast with the model, which starts off from an assumed steady state and acts as if the shock was already absorbed.

5.2.4 Old age dependency



Data sources:
NIPSSR (2017) and Population statistics, census data via e-stat.go.jp:
Ministry of Health, Labour, and Welfare

To finish this section, the old age dependency ratio is evaluated. The old age dependency ratio is defined as the population over 65 years divided by the working age population. This definition is slightly adjusted in the presented model:

instead of using $\frac{POP_{65+}}{POP_{15-64}}$, the model uses $\frac{POP_{70-99}}{POP_{20-69}}$.

This adjustment was needed since the age cohorts in the model are 10 years apart and only start entering the model at age 20.

Luckily, the National Institute of Population and Social Security Research produces statistics in 5-year age groups. As such the reader can immediately compare both graphs.

The predicted old age dependency follows the data closely. This is to be expected as fertility and conditional survival rates were imposed as exogenous forces into the model. One would assume a perfect fit, but certain demographic oddities get averaged. One period in the model equals 10 years and to fit the model one needs to average out the data.

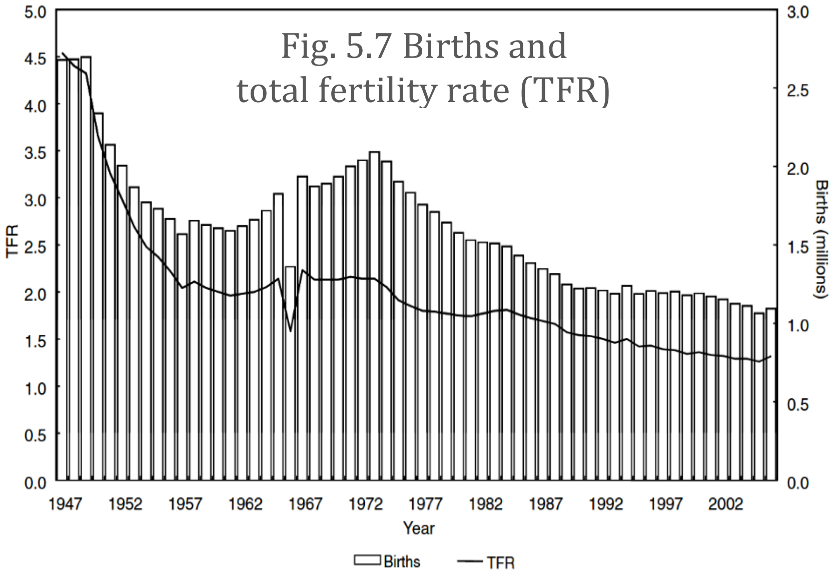


Figure 5.7 is taken from Ogawa et al. (2010); Fig. 4.2 Based on data from Ministry of Health, Labour, and Welfare, Vital Statistics.

The reader can take the example of the 25% drop in the birth-rate in 1966 vis-à-vis 1965. This drop was explained by the year of Fire and Horse (New York Times, 1987)[15] , believed to be a bad omen by many Japanese. To avoid cursing their children, many couples postponed the start of the pregnancy until the year was over[16]. In a 10 year perspective, this has little impact. What goes down in one year, gets compensated in the next year. In the figure below this is clearly exemplified by the birth-rate increase of 40% the following year.

[15] https://www.nytimes.com/1987/01/15/world/japan-s-zodiac-66-was-a-very-odd-year.html

[16] According to zodiac superstition women born in the year of fire and horse will kill their husband when reaching adulthood.

6 Policy and evaluation

In this Chapter the different pension policies that will be evaluated are introduced. The structure of this chapter is as follows: first, the different policy options are introduced. These are briefly discussed and a motivation is given for the chosen policy reforms. Second, the optimality of those policies are assessed. Optimal policy is based on three criteria: (i) macroeconomic performance, (ii) pension finance-ability (i.e. keeping or improving the pension budget) and (iii) improving equity amongst heterogenous agents. In the final section the findings are summarized accompanied by some concluding remarks.

6.1 Pension reforms

To have some societal relevance, one has to choose policies that have been implemented by policymakers or policies that do not receive much academic attention. As an added bonus: "the joys of policy research is the opportunity to shock the bourgeoisie, to point out the hollowness or silliness of official positions" (Krugman, 1993). Another motivation to choose a non-conventional policy field is to point at possible flaws. Indeed, "*The road to Hell is paved with good intentions*" [17]. It is up to policy researchers to examine real-life policies and to suggest implementable alternatives if flaws are found.

Most pension reforms are focussed either on contribution rates or replacement rates. By selecting the following policy shocks this dissertation shifts the discussion from this focus and goes into a more novel way of thinking of pension reform.

Before explaining all the policy reforms, a note is given on the assumptions that is made for the benchmark simulation.

[17] Proverb attributed to Bernard of Clairvaux, French Abbot, XII century.

When policy reforms are simulated they are compared to a benchmark. The benchmark simulation operates under certain assumptions made on the model. For demographics, the benchmark and all policy alternatives follow the NIPSSR (2017) medium fertility, medium mortality projection for 2015 to 2115. As such demographics are allowed to change over time but need to follow the projection set out I NIPSSR(2017).

Regarding labour augmenting technology, the TFP projection from Cette et al. (2017) is used to determining the time series of g_A. Their projection stops in 2100, the model assumes a constant growth rate in labour augmenting technical progress from that point onwards.

In the benchmark simulation, policy variables are assumed to stay at their last known value. In the policy reforms 1 to 5, these policy variables can deviate from their last known value. This is especially true for the consumption tax rate. The reason for this is the compensating mechanism behind each reform: all policies are compensated by a change in the consumption tax rate to assure a fixed debt-to-GDP ratio. The reader can consult table 6.1 to find the values of the policy variables in the benchmark.

All the policy reforms are enforced from 2015 onwards. All the reforms are assumed to be permanent. Because agents have perfect foresight and because the government announces the reform plans in advance, agents already adjust their behaviour in anticipation of the reform. This is a limitation for the policy analysis as these reforms cannot be seen as unanticipated events. In Chapter 7 this limitation to the model is discussed.

6.1.1 Bonus – Malus for late/early retirement

The first policy is one to which Krugman might refer. The bonus-malus system of early/late retirement was introduced by prime minister Abe in 2015. This reform follows in some extent the recommendations of the Belgian *Academische Raad van Pensioenen*. Its similarity might not be clear because of semantics. Since there are differences, it is appropriate to explain the policy from scratch.

The basics of policy 1 has already been explained in section 3.2.2 "Time and budget constraints". In equations (10) and (11), one sees this:

$$ppt_s^t(1 + BMR)^{\kappa(R_s^t - \rho)}$$

The mechanism of the pension bonus lay bare in this expression. The bonus depends on three different aspects: the cut-off point (i.e. the official retirement age, ρ), the rate of the bonus/malus (BMR) and when the employee retires (R_s^t). As a reminder, κ has no impact since it is only a normalization parameter.

Important to note are the limits to the functioning of this bonus-malus system. The retirement window is fixed at 10 years: starting at 60 years old and ending at 70. Between these ages, employees must retire. If they retire before the official retirement age, they get a reduction of their monthly pension payments. If they retire after the official retirement age, they get an additional pension payment. If employees retire at the official retirement age, the bonus is set to zero.

Currently, Japan has an official retirement age of 65 and has set the bonus-malus rate at 7.2% per year of delayed/early retirement[18]. This bonus is paid on the base of all pension earnings and not only on earnings accrued in the last period of active life. As shown above, the Japanese tend to prolong their stay on the labour market: they combine a late exit of the labour market with a reduction in hours worked when growing old. Intuitively this seems like a 'good intentions, bad outcomes' story. For completeness: BMR goes from zero to $BMR = 0.072$ and $\rho = 0.5$.

[18] For completeness, this is the average rate. The current government has set a rate of 8.4% for late retirement and 6% for early retirement.

6.1.2 Shifting accrual rates

The second policy is the first contender to the Bonus-Malus system. Pension finances are clearly in the mind of prime minister Abe. Since the looming default of pension payments, a lot of young workers have lost faith in the government's ability to keep the pension payments going (Takayama,2010)[19].

A way to get the similar effects without increasing the pension liabilities is to alter accrual rates. This policy is taken from Buyse et al (2013, 2017). The accrual rates of young workers are reduced by half and transferred to their last period of active life. Model-wise this translates to: $p_1 = 1/10$ and $p_5 = 3/10$. Other pension weights (p_j) are kept at the original level 1/5. This reform alters the way pension benefits are calculated: see eq. (12.a). As p_5 increases, the return to working in the last period of active life increases and the return the work in the first life decreases. Which will alter the labour-leisure choice of agents in their first and last period of life. An additional reason is the reduction in the cost to education (optimal education FOC) which might push high and medium ability agents into studying more. As a summary:

$$p_1 = 0.1, \qquad p_2 = 0.2, \qquad p_3 = 0.2, \qquad p_4 = 0.2, \qquad p_5 = 0.3$$

6.1.3 Bonus – Malus with increased retirement age

The third policy is a direct improvement of the first policy. Taking into account that the official retirement age is still at 65, one could combine policy one with the condition that the pension age (ρ) is lifted to 67. The reason is quite simple, the bonus system needs to be binding to have an effect. With increased longevity, the most obvious reform is to increase the official retirement age. To make a comparison possible with policy 1 the BMR is kept at 7,2%. In the model this policy translates to: $\rho = 0.7$ and $BMR = 0.072$.

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[19] Pension contributions are not directly deducted from the wage. Many workers have willing defaulted on paying these contribution, increasing the believe that the government cannot sustain the PAYG system.

6.1.4 Bonus only – accrual rates revisited

The fourth policy is a compromise between policy 1 and 2. The malus is completely dropped ($BMR = 0$), but the bonus acts in a similar way as policy one. The difference however is that this policy does not discriminate between late retirement or more hours worked when old. What matters is the total time spent on the labour market when old. In other words, it does not matter if agents increase n_5 by either having more $\tilde{n}_5$ or by delaying retirement (R_s^t), see eq.(12.a).

How does this policy work? There is no shifting of pension weights (p_j) but additional weight is added to the last period of active life: $p5$ is increased with 7.2%-point. This resolves a key problem of policy one; namely that it does not discriminate between working when young and working when old. The bonus in policy one is given to the full set of pension payments. Or put differently, policy one focuses on delaying retirement, but not on increasing n_5. In terms of the model: $p_1 = 0.2$, $p_2 = 0.2$, $p_3 = 0.2$, $p_4 = 0.2$, $p_5 = 0.272$

6.1.5 Combined policies two and three

Lastly, it is interesting to examine the combined effects policies 2 and 3. The reason for this particular combination is that they both have an impact through different channels. Or at least that is what is taken as a first hypothesis. If correct, the overall effect is expected to be stronger than either policy 2 or 3. An additional reason for combining these two policies is that it forms a natural counterexample to policy 4. Policy 4 is a compromise, taking elements from both policy reforms, whilst policy 5 can be seen as an additive. Model-wise the following happens:

$\rho = 0.7$, $BMR = 0.072$, $p_1 = 0.1$, $p_2 = 0.2$, $p_3 = 0.2$, $p_4 = 0.2$, $p_5 = 0.3$

6.1.6 Summary of policies

Table 6.1 Benchmark parameters

$BMR = 0$	$\tau_c = 0.114$	$rr_L = 0.533$
$\rho = 0.5$	$\tau_w = 0.212$	$rr_M = 0.404$
$\boldsymbol{cr_1 = cr_2 = 0.08}$	$\tau_p = 0.153$	$rr_H = 0.355$
$p_1 = p_2 = p_3 = p_4 = p_5 = 0.2$	$\tau_k = 0.202$	$g = 0.393$
$pf_1 = pf_2 = 0.08$		$B/Y = 2.480$

Table 6.2 Summary of policies reforms

	Name	Variable(s) affected
Policy 1	Bonus-Malus for late/early retirement	BMR = 0.072
Policy 2	Shifting accrual rates	p1=0.1 ; p5=0.3
Policy 3	Bonus-Malus with increased retirement age	BMR = 0.072 ; ρ=0.7
Policy 4	Bonus only - accrual rates revisited	p5=0.272
Policy 5	Combined policies two and three	BMR=0.072 ; $\rho = 0.7$; p1=0.1 ; p5=0.3

6.2 Assessment

Table 6.3 Effects of policies 1 to 5 at the start of the policy reform (2015-2024)

	Policy 1	Policy 2	Policy 3	Policy 4	Policy 5
Δn_1	0,180	-0,412	-0,185	0,097	-0,572
Δn_2	0,005	0,013	-0,016	-0,033	-0,003
Δn_3	-0,029	-0,044	0,026	-0,090	-0,018
Δn_4	-0,054	-0,126	0,054	-0,172	-0,070
$\Delta \tilde{n}_5$	0,094	2,545	-0,060	1,731	2,376
Δe	-0,125	0,144	0,107	-0,082	0,238
ΔR	0,007	0,078	0,023	0,052	0,077
Δn_L	0,017	0,389	-0,016	0,256	0,356
Δn_M	0,021	0,301	-0,016	0,216	0,270
Δn_H	0,072	0,076	-0,055	0,158	0,020
ΔR_L	0,008	0,091	0,002	0,061	0,089
ΔR_M	0,006	0,074	0,002	0,050	0,073
ΔR_H	0,005	0,064	0,003	0,044	0,064
Δe_M	0,000	0,039	0,023	-0,017	0,059
Δe_H	-0,002	0,248	0,191	-0,147	0,416
ΔY	-0.730	0.333	0.446	-0.167	0.726
$\Delta(GPP/Y)$	0.932	-0.193	-0.563	0.352	-0.694
$\Delta \tau_c$	0,523	0,148	-0,622	0,400	-0,447
Δr	0,467	0,460	-0,485	0,478	-0,009

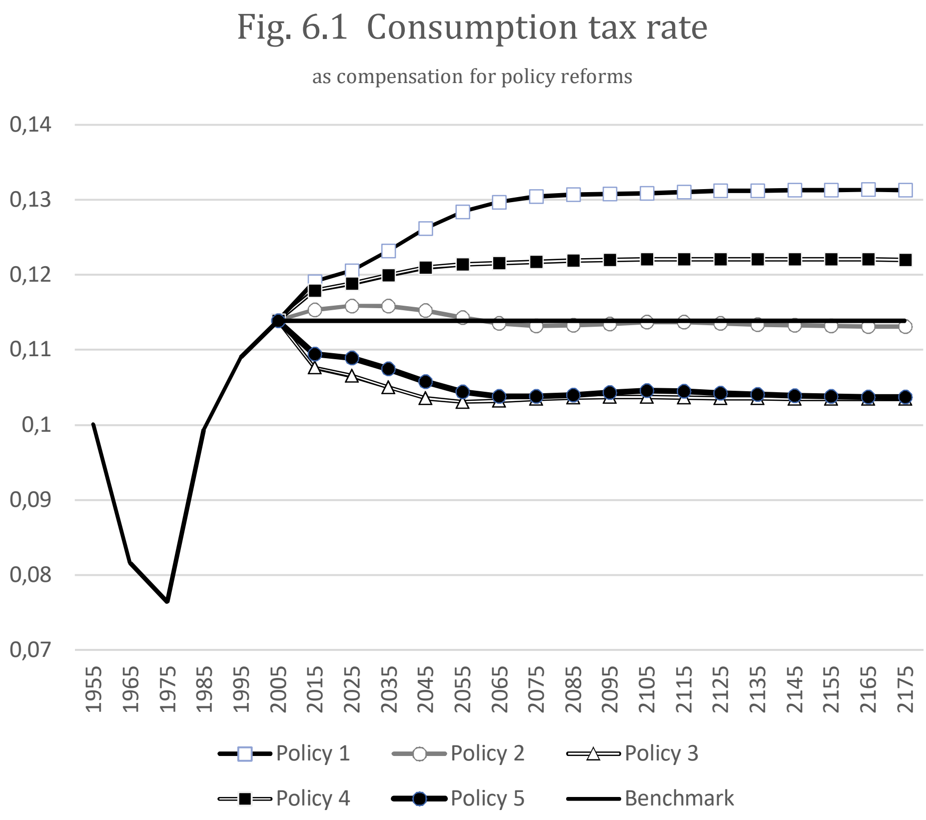
notes: (i) All policies are compensated by a change in the consumption tax-rate to assure a fixed debt-to-GDP ratio.
(ii) All figures are expressed as %-point differences in comparison to the benchmark, with exception of the retirement age (R), which is expressed in years and GDP (Y), which is expressed as a percentage change from the benchmark.
(iii) For hours worked and retirement age, the lack of the ability subscript denotes that it is an averaged effect over all ability groups. Same applies for the absence of age subscripts in hours worked.

Table 6.4 Effects of policies 1 to 5 in the last period of simulation (2175-2184)

	Policy 1	Policy 2	Policy 3	Policy 4	Policy 5
Δn_1	0,261	-0,459	-0,161	0,097	-0,592
Δn_2	0,029	0,010	-0,020	-0,025	-0,009
Δn_3	-0,022	-0,010	0,011	-0,065	0,000
Δn_4	-0,078	-0,036	0,044	-0,128	0,005
$\Delta \tilde{n}_5$	-0,031	2,035	0,061	1,276	2,024
Δe	-0,166	0,170	0,101	-0,082	0,256
ΔR	0,002	0,075	0,055	0,047	0,078
Δn_L	0,009	0,360	0,005	0,212	0,354
Δn_M	0,015	0,282	0,002	0,187	0,273
Δn_H	0,085	0,070	-0,043	0,157	0,028
ΔR_L	0,004	0,088	0,006	0,054	0,091
ΔR_M	0,001	0,073	0,006	0,045	0,075
ΔR_H	0,000	0,063	0,005	0,040	0,065
$\Delta \mathrm{e}_{\mathrm{M}}$	0,000	0,044	0,022	-0,017	0,063
$\Delta \mathrm{e}_H$	-0,003	0,296	0,180	-0,148	0,450
ΔY	-0,772	0,336	0,485	-0,192	0,763
$\Delta(GPP/Y)$	0,927	-0,193	-0,560	0,353	-0,692
$\Delta\tau_c$	1,739	-0,078	-1,044	0,814	-1,020
Δr	0,850	0,238	-0,509	0,526	-0,231

notes: (i) All policies are compensated by a change in the consumption tax-rate to assure a fixed debt-to-GDP ratio.
(ii) All figures are expressed as %-point differences in comparison to the benchmark, with exception of the retirement age (R), which is expressed in years and GDP (Y), which is expressed as a percentage change from the benchmark.
(iii) For hours worked and retirement age, the lack of the ability subscript denotes that it is an averaged effect over all ability groups. Same applies for the absence of age subscripts in hours worked.

In Figure 6.1 the consumption tax rate is plotted over time. Each policy reform needs a mechanism to assure a constant debt-to-GDP ratio. In this dissertation the consumption tax rate was chosen as the compensating policy variable. A different approach would be a compensation through lump-sum transfers that individuals receive. The reason for choosing the consumption tax rate rather than the lump-sum transfers is that lump-sums are more theoretically based and lack practical implementation[20].



[20] The simulations with a lump-sum transfer as compensating mechanism have also been simulated and are available upon request.

6.2.1 Pension financing

Pension finances play a role in the creations of debt. When a pension deficit is created this taken into the government's general budget. This deficit pushes the government to increase the consumption tax rate in order to sustain a constant debt-to-GDP ratio. Reversely, a pension surplus allows the government to decrease the consumption tax rate to maintain a constant debt-to-GDP ratio.

It is this dynamic that will explain why the consumption tax rate decreases in policies 3 and 5 and increases in policies 1, 2 and 4.

6.2.2 Macroeconomic performance

Looking at macroeconomic performance, we see the clear negative effect of policy 1 on output. Growth will be lower in comparison to the benchmark. The reasoning behind it is simple and straightforward: a bonus-malus system has positive macroeconomic effects when the malus rate is binding. Most Japanese postpone retirement as a default (see chapter 1). When policymakers introduce a bonus-malus system without controlling for the effective retirement age, the result is counterproductive. In essence, the majority of households are entitled to the pension bonus. This in turn changes the incentives to work and to study. Agents see their lifetime income increase, this gives rise to more (future) consumption, which alters the labour-leisure choice that agents make. But at the same time the rate on consumption taxes is increased. This alters the Euler equation: the increasing path of consumption tax rates makes agents shift consumption from the future to now. This creates a shortage on the capital market. Higher interest rates are needed for the supply to follow. These higher returns push agents to postpone some of their consumption. As a result the markets clear at a higher rate. The higher consumption tax rate also pushes agents to work more: the increase in taxes lowers the consumption of agents and makes them revaluate the labour-leisure choice. The result is an increase in work, which also comes at the cost of fewer time spent in education. Lastly the effects are different between ability types. This can be explained by the consumption tax rate. A higher τ_c affects the high ability households more as they consume more than lower ability households.

Policy 1 is in clear contrast with policy 3, where the retirement age is binding. The same dynamics are at play as policy 1 but now agents face the negative income effect of the Bonus-Malus system.

Most households now receive a deduction in pension payments. Agents see their lifetime income decrease. This diminishes their future consumption possibilities and as result they will supply more work in order to attain optimal consumption. The compensating tax rate goes down which changes the consumption possibilities of agents in the reversed direction. Suddenly agents can consume more and as a result will revaluate the labour-leisure choice in favour of more leisure. Again the tax rate also impacts the Euler equation. A decreasing path of consumption tax rates pushes agents to shift consumption to later periods of life. This has positive effects on aggregate wealth, which creates a surplus on the capital market. Firms will react and install more capital but at lower interest rates. The lower interest rate makes households provide fewer assets to the capital market. This goes on until eq. (22) is restored

Lastly, high and medium able agents see their return to human capital increase and as a result their optimal level of education will be higher than in the benchmark.

Concerning policy 2, The shifting of accrual rates has positive macroeconomic effects. Albeit smaller than policy 3. What is clear is that the weight of adjustment mainly lie with the lower ability groups. Low ability households increase hours worked and postpone retirement more than the high ability households. There is also a clear shift in hours worked over the life cycle. As agents see their return to work in the 1st period of life decrease and the return to work in the 5th period of life increase, they will shift accordingly. In short, agents work less when young and more when old. In this aspect policy 2 is a clear winner. It activates older workers more than any other policy option. Because earning in the last period of life are generally speaking higher than earning earlier in life, the government needs to pay out more pension benefits. This results in an increase of the consumption tax rate over time. The effects on the Euler equation and the interest rate are similar to policy 1 but of a much smaller magnitude. In the long run this channel even dies out (see Fig. 6.1). The resulted increase in the retirement age will also push high and medium ability households to spend more time on education.

Examining policy 4, we do see more labour participation amongst the old and at the same time they postpone retirement. Like policy 1 this comes at a great cost. This bonus is related to the last period of work but is given to all households regardless of the retirement age. Agents see their lifetime income increase, this gives rise to more (future) consumption, which alters the labour-leisure choice that agents make. They will want to enjoy more leisure. Unfortunately this costly reform will create an increased path on the consumption tax rate. The impact through the Euler equation and the interest rate are again similar to policy 1: the increasing path of consumption tax rates makes agents shift consumption from the future to now. This creates a shortage on the capital market. Higher interest rates are needed for the supply to follow. These higher returns push agents to postpone some of their consumption. As a result the markets clear at a higher rate. The consumption tax rate also pushes agents to work more: the increase in taxes lowers the consumption of agents and makes them revaluate the labour-leisure choice. The net result is an increase in work and this at the cost of time spend on education.

Lastly, policy 5 shows how the best of policy 2 and 3 are combined. The negative lifetime income effect decreases (future) consumption possibilities. This channel is fully analogous to policy 3 and will not be repeated. The interest rate channel is different. The market clears at a lower interest rate in comparison to the benchmark. This leads investments to increase, resulting to an increase of the capital stock and that increases the labour productivity of all workers. The shifting of accrual rates will also decrease the cost to education for high and medium ability individuals. They will spend more time on education and this again increases the labour productivity of these agents. This is the effect originally attributed to policy 2.

6.2.3 Welfare effects

Unfortunately good macroeconomic performance does not guarantee an increase in welfare. For starters, there are transitional effects: a situation can get worse before it gets better. Secondly, the effects need not be uniform for all agents. There can be winners and losers from economic growth. Taking the argument the other way around: some losses can be accepted if the gains accrue to those who are worst off. Lastly, welfare comes in different forms. Some might enjoy the extra leisure even at the cost of some consumption.

In order to analyse these trade-offs, I use a welfare analysis taken from Buyse et al(2017). Welfare is measured as the constant percentage change in benchmark consumption in each period of remaining life that households should get to attain the same lifetime utility as after the policy reform. This method allows me to easily plot the combined effect of additional utility due to consumption and leisure for each policy.

The graphs plotted below are quite unique : the horizontal axis goes from k=-7 to k=7, representing the different generations that are currently alive and those who still need to be born at the time of the shock. The index k refers to the historic time in which generations are born relative to the time the policy shock first occurs. E.g.: the current age cohort 20-29 will be plotted at k=0. Whilst on the vertical axis the percentage change in lifetime consumption is plotted which serves as my welfare measure.

Fig. 6.2 Welfare for low ability

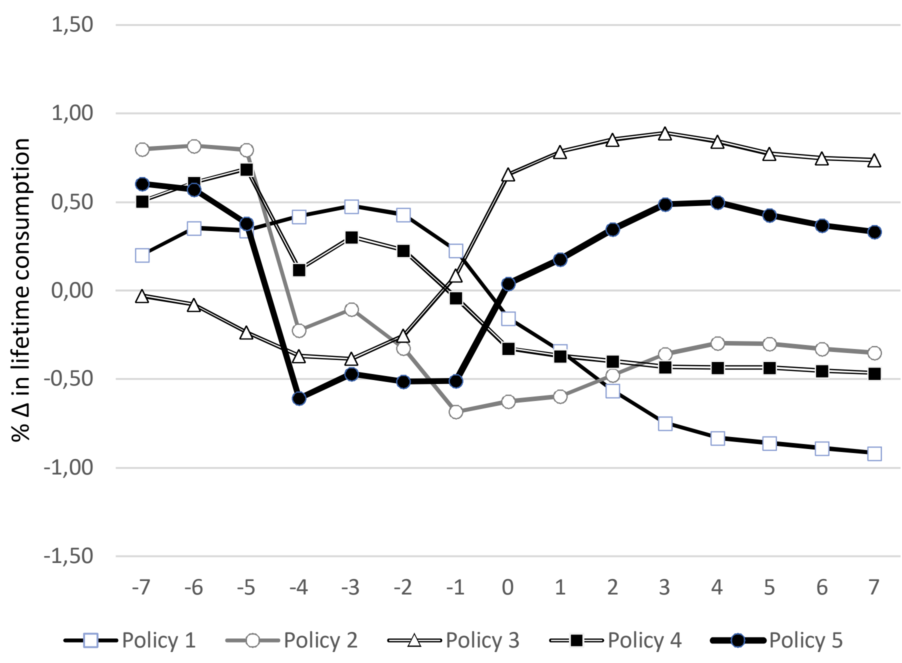


Fig. 6.3 Welfare for medium ability

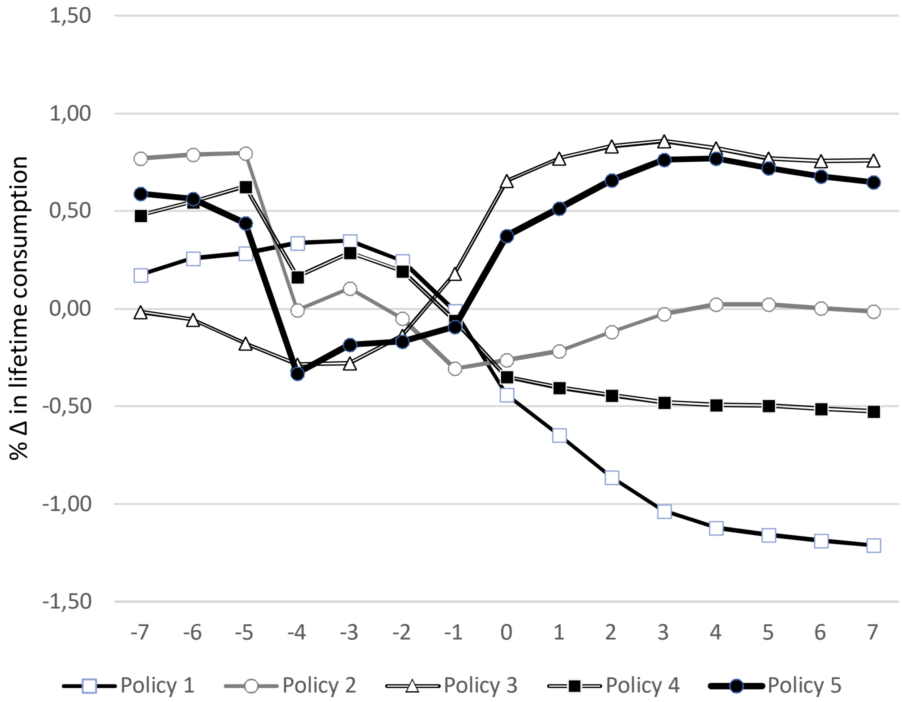
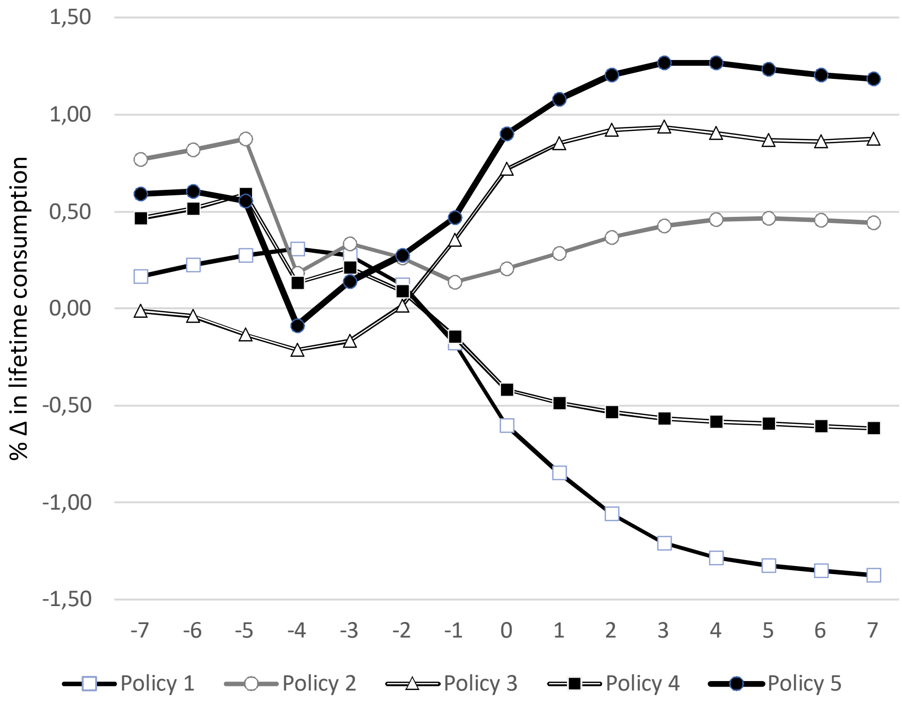


Fig. 6.4 Welfare for high ability



I separated the welfare graphs by ability type. To make a sensible comparison across ability types, I scaled all three graphs such that a direct comparison is possible[21]. The reason for this configuration is that I can clearly determine the optimal policy for each ability group. Next to ability, we can also easily separate the effect of current generations (k=-7 to k=0) and future generations (k=1 to k=7).

Looking over all ability types we see a clear distinction between what's beneficial for the current generation and what creates welfare for their offspring. Broadly speaking policies 1 , 2 and 4 create welfare gains for the current generations at the cost of welfare for the future generations. Reversely policies 3 and 5 create mainly welfare gains for the future generations.

A second broad distinction is between ability types. The effects on welfare are the largest for the high ability households and are the smallest for the low ability households. This effect is more pronounced for future generations. The reason is that a consumption tax rate affects higher incomes more than lower incomes because these higher incomes consume more.

Immediately the tragedy of the policy choice becomes clear. If the current generations chose optimal, they doom the future generations to a large welfare loss. Reversely, if the current generation choses the policy that makes them worst off, the future generations reap all the benefit.

Next to the choice between generations, one ought to look at the disperse effects amongst ability. Take for example policies 3 and 5. Both help future generations, but policy 5 is more catered to the high ability, whilst policy 3 is more suited to the low and medium ability households. Policy makers could therefore prefer policy 3 over policy 5. In general high ability households tend to have higher levels of welfare. Policy 3 could therefore be classified as a social measure.

It becomes clear that no policy measure wins on all three criteria of optimal policy.

[21] Alternatively, I suggest that the interested reader consult appendix C.2.1 to find the same graphs but split by policy instead of ability.

7 Limitations of the model

7.1 Shortcomings

In this chapter the shortcoming of the model are briefly discussed. These shortcomings can serve as opportunities for future research.

7.1.1 Exogenous growth

The model described in Chapter 3 prohibits the transfer of human capital from one generation to the following generation. A possible extension could be to allow the transfer of human capital stock. In doing so the model would enter group of endogenous growth model. It could be reasonably assumed that behaviour and aggregate outcomes will be different.

7.1.2 Streamlined pension system

The first pillar pension system of this model does not represent the real Japanese pension system in full. Two parts are missing that might enrich future models. First the Bonus-Malus Rate is not equal in both direction. The malus rate is slightly smaller than the bonus rate. A new equation that makes a difference between these two rates is desirable. Secondly, the model assumes universal coverage of the first pension pillar. In reality access to the first pension pillar is conditional on individuals having paid 20 years of contributions when still at work. One could model this in the budget constraints of agents. Or a drop-out rate in pension benefits could be introduced to cover the partial coverage.

7.1.3 Other drivers of demography

As mentioned before, the third driver of demographic change is the net migration rate. Whilst at the moment this seems not to be an issue for Japan, this could be important if we model demographic change in other countries. The United States has always been seen as a country of immigrants, but in recent years this has changed. In demographic terms this alters the net flow migration. The change in migration on its turn will affect the demographic makeup of society.

7.1.4 Gender

A last possible extension is to make the model more heterogenous by adding gender values to agents. In the last 50 years the entry of woman onto the labour market has been astounding. In the case of Japan this has not happened in the same extent. The inclusion of gender in the model would allow more complexity and assert if policy have differential effects across genders.

7.1.5 Unanticipated reforms

Whilst this is not an extension, this is a shortcoming that needs to be addressed in future versions of this model. At the moment to code underlying the model can only handle anticipated policy reforms. In future research it would be advised to change the coding of the model to allow for unanticipated events.

8 Conclusion

In this dissertation a large scale 8-period OLG model with demographic change and 3 ability types was built and validated. This allowed to assess policy reforms that are either implemented at this moment (policy 1) or reforms that are academically interesting (policies 2 to 5).

In the first chapter the broad outlines of the dissertation were described, introducing the reader to the big parts of the model. In the second chapter a non-technical overview of the model was presented. In Chapter three the technicalities of chapter two were explained in full and the equations behind the model were presented. Chapter four explained how the parameters of the model were assigned a value. Some were assigned a value by referring to the literature, whilst others were determined through calibration. The procedure for calibration, based on Ludwig et al. (2012) was explained in 6 simple steps. At the end chapter 4 the result from this calibration effort were presented in Table 4.1. Chapter 5 validated the model by comparing key macroeconomic variables to real data. This backfitting exercise resulted in successfully predicting the past. In Chapter 6 policy reforms were introduced and evaluated. The evaluation criteria were: (i) macroeconomic performance, (ii) pension finance-ability (i.e. keeping or improving the pension budget) and (iii) improving equity amongst heterogenous agents. Criteria (i) was done by looking at growth, education, effective retirement age and hours worked. Criteria (ii) was assessed by looking at the changes in the pension budget deficit (GPP_t). Lastly criteria (iii) was evaluated by looking at the welfare measure from Buyse et al.(2017). The welfare of heterogenous agents were examined, with a focus on ability type and the different generations.

Chapter 7 reflected on the model that was used and links the shortcomings of the model to possible extensions. These extensions include: human capital accumulation across generations, heterogenous agents by gender and migration as a driver of demography. A final suggestion was the alteration of the coding of the model in order to simulate unanticipated reforms.

Concerning the policy reforms, this dissertation showed the negative consequences on economic growth for policy 1. Other more suitable policy reforms were investigated. Unfortunately no alternative won on all the criteria of optimal policy. In general, the low ability individuals and the high ability individuals have different optimal policies. Similarly optimal policy differs for the current versus the future generations. As a result policy makers are advised to weigh the benefits and the costs that differentiate across agents and time. A possible solution would be to see the pension reform in a broader fiscal system. This leaves policymakers with more possibilities to counter the negative effects of certain pension reforms.

References

Auerbach, A., Hagemann, R., Kotlikoff, L. and Nicoletti, G. (1989). The Economic Dynamics of an Ageing Population: The Case of Four OECD Countries. *OECD Economics Department Working Papers*, No. 62.

Barro, R. (1990). Government spending in a simple model of endogenous growth. *Journal of Political Economy*, 98:103–125.

Barro, R. and Lee, J. (2013). A new data set of educational attainment in the world, 1950-2010. *Journal of Development Economics*, 104:184–198.

Bouzahzah, M., de la Croix, D. and Docquier, F. (2002). Policy reforms and growth in computational OLG economies, *Journal of Economic Dynamics and Control*, 26, 2093–2113.

Buyse, T., Heylen, F., and Van de Kerckhove, R. (2013) Pension reform, employment by age, and long-run growth. *Journal of Population Economics*, 62, 769–809.

Buyse, T., Heylen, F., and Van de Kerckhove, R. (2017). Pension reform in an OLG model with heterogeneous abilities. *Journal of Pension Economics and Finance*, 16, 144–172.

Caselli, F., Coleman, I., & John, W. (2006). The world technology frontier. *American Economic Review*, 96(3), 499–522.

Cette, G., Lecat, R. and Ly-Marin, C. (2017): "Long-term growth and productivity projections in advanced countries," *Banque de France*. Working paper No. 617

Cournède, B. and Gonand, F. (2006). Restoring fiscal sustainability in the euro area: raise taxes or curb spending. *OECD Economics Department Working Papers.* No. 270.

de la Croix, D. & Michel, P. (2002). *A theory of economic growth: dynamics and policy in overlapping generations*. Cambridge University Press.

Devriendt, W., & Heylen, F. (forthcoming). Macroeconomic and distributional effects of demographic change in an open economy - The Case of Belgium*. Journal of Demographic Economics.*

Diamond, P. A. (1965). National debt in a neoclassical growth model. *The American Economic Review*, 55(5), 1126–1150.

Docquier, F. & Paddison, O. (2003) Social security benefit rules, growth and inequality. *Journal of Macroeconomics*, 25, 47–71.

Feenstra, R. C., Inklaar, R., and Timmer, M. P. (2015). The next generation of the Penn World Table. *American Economic Review*, 105(10), 3150–3182

Goodhart, C & Erfurth, P. (2014) Demography and economics: Look past the past. *VOXEU.org*, 4 November.

Heylen, F. & Van de Kerckhove, R. (2019). Getting low educated and older people into work: The role of fiscal policy. *Journal of Policy Modeling,* 41(4): 586-606.

Heylen, F. & Van de Kerckhove, R. (2013). Employment by age, education, and economic growth: effects of fiscal policy composition in general equilibrium. *The B.E. Journal of Macroeconomics*, 13(1), 49–103.

Krueger, D. & Ludwig, A. (2007). On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare. *Journal of Monetary Economics*, 54:49–87.

Krugman, P. (1993). How I work. *The American Economist*, 37 (2): 25-31.

Lucas, R. (1990). Supply-side economics: an analytical review. *Oxford Economic Papers*, 42:292–316.

Ludwig, A., Schelkle, T., and Vogel, E. (2011). Online appendix to ”demographic change, human capital and welfare”. *Review of Economic Dynamics*. Technical Appendices 08-168.

Muto, I., Oda, T., and Sudo, N. (2012). *Macroeconomic Impact of Population Aging in Japan: A Perspective from an Overlapping Generations Model.* (Bank of Japan Working Paper Series No. 12-E-9). Bank of Japan. Tokyo, Japan.

NIPSSR (2017). *Population Projections for Japan (2017): 2016 to 2065.* Department of Population Dynamics Research, National Institute of Population and Social Security Research. Tokyo, Japan.

Ogawa, N., Mason A., Chawla A., and Matsukura, R. (2010). Japan's Unprecedented Aging and Changing Intergenerational Transfers. In Ito, T. and Rose, A. (eds). *The Economic Consequences of Demographic Change in East Asia*. University of Chicago Press.

OECD (2016), *OECD Territorial Reviews: Japan 2016*, OECD Territorial Reviews, OECD Publishing, Paris,

OECD (2018), *International Migration Outlook 2018*, OECD Publishing, Paris

Onder, H. & Pestieau, P. (2014). Is aging bad for the economy? Maybe. *Economic Premise*, 144. The World Bank.

Rogerson, R. (2007). Taxation and market work: is Scandinavia an outlier? *Economic Theory*, 32(1):59–85.

Rogerson, R. and Wallenius, J. (2009). Micro and macro elasticities in a life cycle model with taxes. *Journal of Economic Theory*, 144:2277–2292.

Samuelson, P. A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of Political Economy*, 66(6), 467–482.

Sommacal, A. (2006). Pension systems and intragenerational redistribution when labor supply is endogenous. *Oxford Economic Papers*, 58:379–406.

Takayama, N. (2010) Pension Issues in Japan: How Can We Cope with the Declining Population? In Ito, T. and Rose, A. (eds). *The Economic Consequences of Demographic Change in East Asia*. University of Chicago Press.

United Nations (2017). World Population Prospects: The 2017 Revision. *Department of Economic and Social Affairs*, *Population Division*.

Valdés, B. (2003). An Application of Convergence Theory to Japan's Post-WWII Economic "Miracle". *The Journal of Economic Education, 34*(1), 61-81.

A First Order Conditions

Appendix A shows the different equations that represent optimizing behaviour for households. First come the Euler equations regarding consumption, followed by the Labour-Leisure choice that agents make. Thirdly the optimal retirement age. Lastly, optimal education.

A.1 Consumption

$$\frac{c_{J+1,s}^{t}(1+\tau_c)}{c_{J,s}^{t}(1+\tau_c)} = \beta s r_J^t (1 + r_{t+J})$$

The Euler equation takes on a typical form, with the exception of the survival rate sr_J^t that is added. One can interpret this just like one would interpret the time preference β. The known probability of dying impacts one's choice to shift consumption from this period to the next. If there is a high probability to die, one will prefer to consume now, rather than later. But If survival rates are rising (i.e. longevity), more consumption will shift to the next period.

Another result is that with age, survival rates drop. As a result the consumption path will become flatter as an individual ages (Devriendt & Heylen, 2017). This can also been shown be looking at RHS of the Euler equation. When $sr_j^t < \frac{1}{\beta(1+r)}$, the consumption path turns negative.

A.2 Labour-Leisure

Agents will spend time working until the discounted marginal utility of labour equalizes that of leisure. The choice they make in period i has an impact on leisure and consumption in period i, but also on consumption at age 5, 6, 7 and 8. This happens because work also increases pension benefits, and thus future consumption possibilities.

Periods 1 - 4 (ages 20-59)

$$\beta^{i-1}\pi_i \frac{\gamma_i}{l_i^{\theta}} \frac{-\partial l_i}{\partial n_i} = \left[\frac{\beta^{i-1}\pi_i}{c_i^t(1+\tau_c)} \left(w_{t+i-1}^s \varepsilon_i h_{i,s}^t (1-\tau_w - pf_1) \right) \right.$$
$$+ \frac{\beta^4 \pi_5}{c_5^t(1+\tau_c)} \left((1-R_s^t)(1+BMR)^{\kappa(R_s^t-\rho)} \frac{\partial ppt_s^t}{\partial n_i} + \frac{\partial spp_s^t}{\partial n_i} \right)$$
$$\left. + \sum_{j=6}^{8} \frac{\beta^{j-1}\pi_j}{c_j^t(1+\tau_c)} (1+BMR)^{\kappa(R_s^t-\rho)} \frac{\partial ppt_s^t}{\partial n_i} \right]$$

With:

$$\frac{\partial ppt_s^t}{\partial n_i} = rr_s(p_i w_{t+i-1} \varepsilon_i h_i (1-\tau_w)) \prod_{j=i}^{4} wg_{t+j}$$
$$\frac{\partial spp_s^t}{\partial n_i} = \frac{\partial credit_i^t}{\partial n_i} \prod_{j=i}^{4} (1+r_{t+j})$$
$$\frac{\partial credit_i^t}{\partial n_i} = (pf_1 + pf_2) w_{t+i-1} h_i^t \varepsilon_i$$

Where BMR stands for the Bonus Malus Rate that the government sets for late/early retirement.
Where wg stands for the annual valorisation rate of previous earnings.
Where pf_1 and pf_2 stand for the contribution rate to the 2nd pillar fund by workers and firms respectively.

In the last period of active life, the labour-leisure choice, takes on a different form. This is because of the split of the period in two fractions : one fraction of active life (R) and one during retirement (1-R). But the rational is identical to previous periods. Agents will shift time spent between working and leisure until the discounted marginal utilities are equalized.

Period 5 (ages 60-69)

$$\beta^4\pi_5\frac{\gamma_5}{l_5^{\theta}}\frac{-\partial l_5}{\partial \tilde{n}_5} = \left[\frac{\beta^4\pi_5}{c_5^t(1+\tau_c)}\left[(1-R_s^t)(1+BMR)^{\kappa(R_s^t-\rho)}\frac{\partial ppt_s^t}{\partial \tilde{n}_5}+\frac{\partial spp_t}{\partial \tilde{n}_5}\right.\right.$$

$$\left.\left.+R_s^t\left(w_{t+4}^s\varepsilon_5 h_{5,s}^t(1-\tau_w)\right)\right]+\left[\sum_{j=6}^{8}\frac{\beta^{j-1}\pi_j}{c_j^t(1+\tau_c)}(1+BMR)^{\kappa(R_s^t-\rho)}\frac{\partial ppt_s^t}{\partial \tilde{n}_5}\right]\right]$$

With:

$$\frac{\partial ppt_s^t}{\partial \tilde{n}_5} = rr_s p_5 w_{t+4}\varepsilon_5 h_5 R_s^t(1-\tau_w)$$

$$\frac{\partial spp_s^t}{\partial \tilde{n}_5} = \frac{\partial credit_5^t}{\partial \tilde{n}_5}$$

$$\frac{\partial credit_5^t}{\partial \tilde{n}_5} = (pf_1+pf_2)w_{t+4}h_5^t\varepsilon_5 R_s^t$$

$$\frac{-\partial l_5}{\partial \tilde{n}_5} = Z\left(\mu R_s^t(1-\tilde{n}_5)^{1-\frac{1}{\zeta}}+(1-\mu)(1-R_s^t)^{1-\frac{1}{\zeta}}\right)^{\frac{1}{\zeta-1}}\mu\left(R_s^t(1-\tilde{n}_5)\right)^{1-\frac{1}{\zeta}}R_s^t$$

A.3 Optimal Retirement Age

Agents shift the retirement age until the discounted marginal benefits equalize the marginal cost of the retirement age. The cost is seen on the LHS, whilst the benefits are seen on the RHS.

$$\beta^4 \pi_5 \frac{\gamma_5}{l_5^{\theta}} \frac{-\partial l_5}{\partial R_s^t} = \Bigg[\frac{\beta^4 \pi_5}{c_5^t (1+\tau_c)} \big[ppt_s^t \, (1+BMR)^{\kappa(R_s^t-\rho)}(-1) + (1-R_s^t)(1+BMR)^{\kappa(R_s^t-\rho)} \, rr_s p_5 w_{t+4} \varepsilon_5 h_5 \tilde{n}_5 (1-\tau_w) + (1-R_s^t) ppt_s^t (1+BMR)^{\kappa(R_s^t-\rho)} \ln(1+BMR)\, \kappa \; + w_{t+4}^s \varepsilon_5 h_5 \tilde{n}_5 (1-\tau_w) + (pf1+pf2) w_{t+4}^s \varepsilon_5 h_5 \tilde{n}_5 \big] + \big[(1+BMR)^{\kappa(R_s^t-\rho)} rr_s p_5 w_{t+4} \varepsilon_5 h_5 \tilde{n}_5 (1-\tau_w) + \; ppt_s^t (1+BMR)^{\kappa(R_s^t-\rho)} \kappa \ln(1+BMR) \big] \sum_{j=6}^{8} \frac{\beta^{j-1} \pi_j}{c_j^t (1+\tau_c)} \Bigg]$$

with:

$$\frac{-\partial l_5}{R_s^t} = Z \left(\mu \big(R_s^t (1-\tilde{n}_5) \big)^{1-\frac{1}{\zeta}} + (1-\mu)(1-R_s^t)^{1-\frac{1}{\zeta}} \right)^{\frac{1}{\zeta-1}} \left((1-\mu)(1-R_s^t)^{-\frac{1}{\zeta}} - \mu \big(R_s^t (1-\tilde{n}_5) \big)^{-\frac{1}{\zeta}} (1-\tilde{n}_5) \right)$$

A.4 Optimal education

$$\frac{\gamma_1}{l_1^{\theta}}\frac{-\partial l_1}{\partial e^t} - \frac{1}{(1+\tau_c)c_1^t}$$
$$= \frac{1}{1+\tau_c}\left[\sum_{j=2}^{4}\frac{\beta^{j-1}\pi^j}{c_j^t}w_{t+j-1}^s\varepsilon_j n_j^t(1-\tau_w-pf_1)\frac{\partial h_j^t}{\partial e^t}\right.$$
$$+\frac{\beta^4\pi^5}{c_5^t}\left((1-R_s^t)(1+BMR)^{\kappa(R_s^t-\rho)}\frac{\partial ppt_s^t}{\partial e^t}+\frac{\partial spp_t}{\partial e^t}\right.$$
$$\left.+R_s^t\left[w_{t+4}^s\varepsilon_5\tilde{n}_{5,s}^t(1-\tau_w-pf_1)\right]\frac{\partial h_j^t}{\partial e^t}\right)$$
$$\left.+(1+BMR)^{\kappa(R_s^t-\rho)}\frac{\partial ppt_s^t}{\partial e^t}\sum_{j=6}^{8}\frac{\beta^{j-1}\pi^j}{c_j^t}\right]$$

With :

$$\frac{\partial h_j^t}{\partial e^t} = h_1^t\,\sigma\phi_s(e^t)^{\sigma-1}$$

$$\frac{\partial ppt_s^t}{\partial e^t} = rr_s(1-\tau_w)(p_2w_{t+1}^s\varepsilon_2n_{2,s}^t wg_{t+2}wg_{t+3}wg_{t+4} + p_3w_{t+2}^s\varepsilon_3n_{3,s}^t wg_{t+3}wg_{t+4}$$
$$+p_4w_{t+3}^s\varepsilon_4n_{4,s}^t wg_{t+4} + p_5w_{t+3}^s\varepsilon_5R_s^t\tilde{n}_s^t)\frac{\partial h_j^t}{\partial e^t}$$

$$\frac{\partial spp_t}{\partial e^t} = \sum_{j=2}^{4}\frac{\partial credit_{j,s}^t}{\partial e^t}\prod_{i=j}^{4}(1+r_{t+i}) + \frac{\partial credit_{5,s}^t}{\partial e^t}$$

$$\frac{\partial credit_{j,s}^t}{\partial e^t} = (pf_1+pf_2)w_{t+j-1}^s\varepsilon_j n_{j,s}^t\frac{\partial h_j^t}{\partial e^t}$$

$$\frac{\partial credit_5^t}{\partial \tilde{n}_5} = (pf_1+pf_2)w_{t+4}h_5^t\varepsilon_5R_s^t$$

$$\frac{\partial spp_t}{\partial e^t} = h_1^t\,\sigma\phi_s(e^t)^{\sigma-1}(pf_1+pf_2)\left(w_{t+1}^s\varepsilon_2n_{2,s}^t(1+r_{t+2})(1+r_{t+3})(1+r_{t+4})\right.$$
$$+w_{t+2}^s\varepsilon_3n_{3,s}^t(1+r_{t+3})(1+r_{t+4}) + w_{t+3}^s\varepsilon_4n_{4,s}^t(1+r_{t+4})$$
$$\left.+w_{t+4}^s\varepsilon_5R_s^t\tilde{n}_{5,s}\right)$$

B Data: sources and construction

Appendix B summarizes the sources and the computations in creating the variables of the model. As the model follows Devriendt and Heylen (forthcoming) in great extent, the reader is often referred to their paper. Where methodology or sources differ this is explicitly mentioned.

B.1 Employment, education and growth

Employment rate in the different age groups (n_1, n_2, n_3, n_4, n_5):
Devriendt and Heylen (Fortcoming)'s method is used. Data sources however are different. Due to the lack of Eurostat data I refer to the labour force statistics from the Japanese e-stat (e-stat.co.jp) which offers employment rates in persons by age group. Some of these data files are in Japanese, but descriptions in English can be found either in the exported data files or in on the e-stat website.

The average effective retirement age (R^t):
The reader is referred to Buyse et al (2019). Data source remains OECD, Ageing and Employment Policies – Statistics on effective age of retirement

The education rate of the young (e_M and e_H):
The reader is referred to Devriendt and Heylen (Fortcoming).

Annual real GDP growth rate:
Data source is OECD, Economic outlook 104.

B.2 Policy variables

Tax rate on consumption (τ_c):

Data source is McDaniel (2007, updated 2014). Averaged rates over 10 years.

Tax rate on labour income (τ_w):

The reader is referred to Buyse et al (2019).

Data source: OECD, Statistical Compendium, Financial and Fiscal Affairs, Taxing Wages, Comparative tax rates and benefits

Tax rate on capital (τ_k):

Data source is McDaniel (2007, updated 2014). Averaged rates over 10 years.

Government spending as fraction of GDP (g):

Data source is IMF, Fiscal Prudence and Profligacy database.

Debt as fraction of GDP (B/Y):

Data source is IMF, Historic debt database.

Net pension replacement rates (rr_s):

Data sources are OECD, Pension at glance (2005,2007,2009,2013, 2017) and (Nugochi 1983) for the oldest replacement rates. For these replacement rates only averages are given. To compute the respective ability related replacement rates (rr_L, rr_M and rr_H) the first known proportion between these different rates was used to extrapolate the series.

Other pension parameters (ρ , BMR, p_j, cr):

Data sources are OECD pension at glance (2005,2007,2009,2013, 2017).

B.3 Exogenous drivers of the model

Fertility rate (f_t):

The reader is referred to Devriendt and Heylen (Forthcoming), but note the different use of periods. These authors use 3 year periods, whilst this dissertation uses 10-year periods.

Data source is the National Institute of Population and Social Security Research, historic population statistics, population by 5 year age group.

Survival rates (sr_j^t):

Are calculated as 1 minus the mortality rate within that age cohort.

Data source is the National Institute of Population and Social Security Research, mortality tables, by 10 year age group.

Labour augmenting technical progress (g_a):

The reader is referred to Devriendt and Heylen (Forthcoming).

Data sources is Penn World Table 8.1 and Cette, G., Lecat, R. and Ly-Marin, C. (2017).

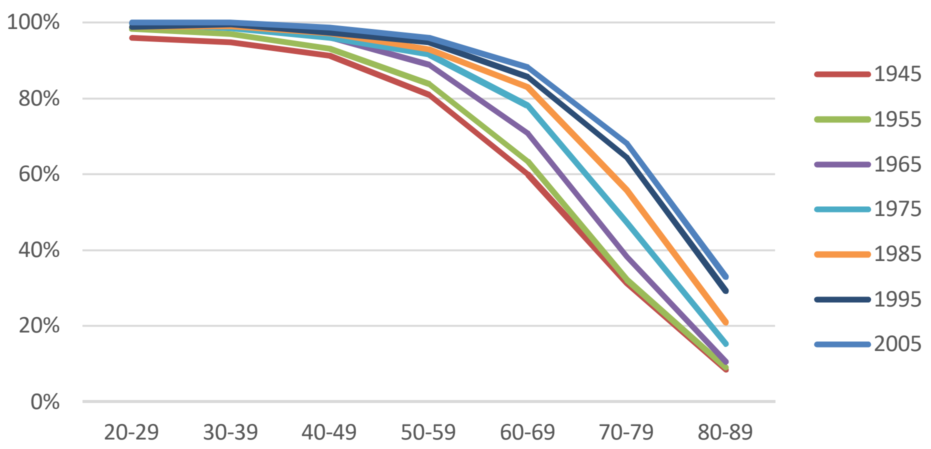
C Additional Graphs

In Appendix C the reader finds the additional plots that are referred to in the main text. The plots are ordered by appearance in the main text.

C.1 Chapter 5

C.1.1 Conditional Survival rates

Fig. C.1 Evolution of survival rates



This graph plots the conditional survival rates[22] on the vertical axis and the age group on the horizontal axis. The different curves represent the different periods in time. For the colour-impaired reader; the inner to outer graphs are chronologically ordered from 1945 to 2005. How should the reader interpret this? The more distance between the different curves (10-years apart), the more conditional survival rates have increased in that period.

Two things become clear: (i) mortality is mostly situated at the tail and (ii) improvements to longevity mostly happen in the second half of the life-cycle.

[22] The survival rates should be interpreted as the percentage of the population in that age group surviving the following ten years.

C.2 Chapter 6

C.2.1 Alternative welfare graphs

These graphs belong to Chapter 6, section 6.2.3 Welfare effects. These graphs are identical to Figures 6.2 to 6.4. The reason for appending them, is to facilitate ease of reading. The splitting of the welfare graphs per policy instead of per ability group allows an easier way to compare the effects that each policy has on the different ability households.

Fig. C.2 Policy 1

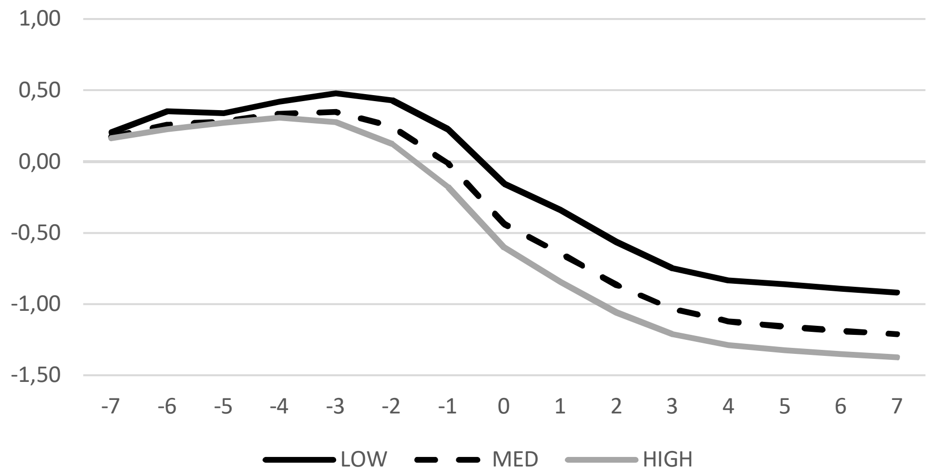


Fig. C.3 Policy 2

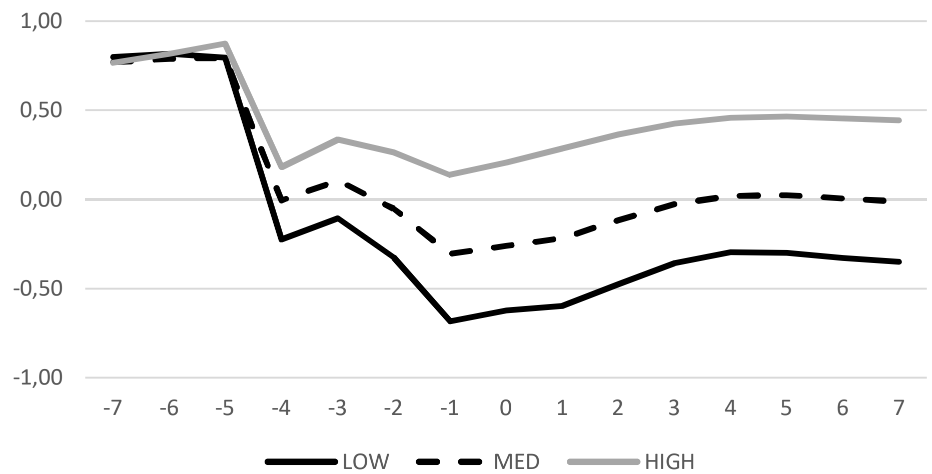


Fig. C.4 Policy 3

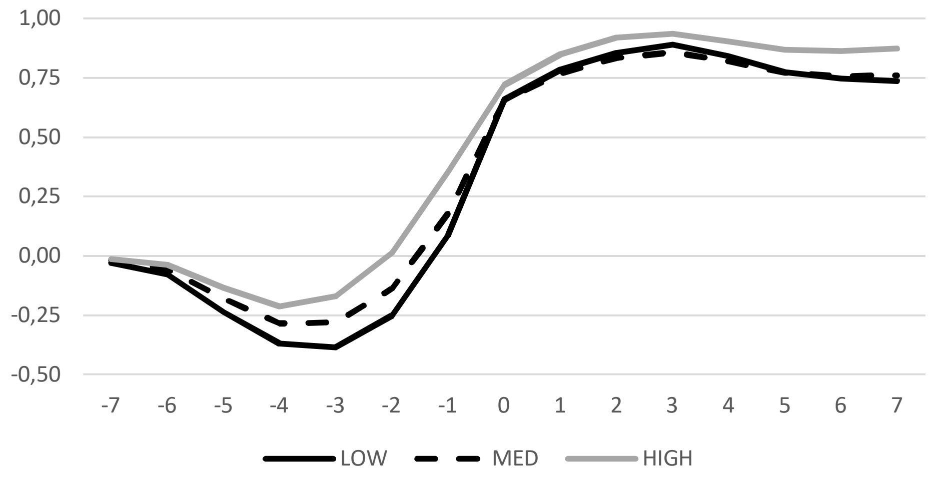


Fig. C.5 Policy 4

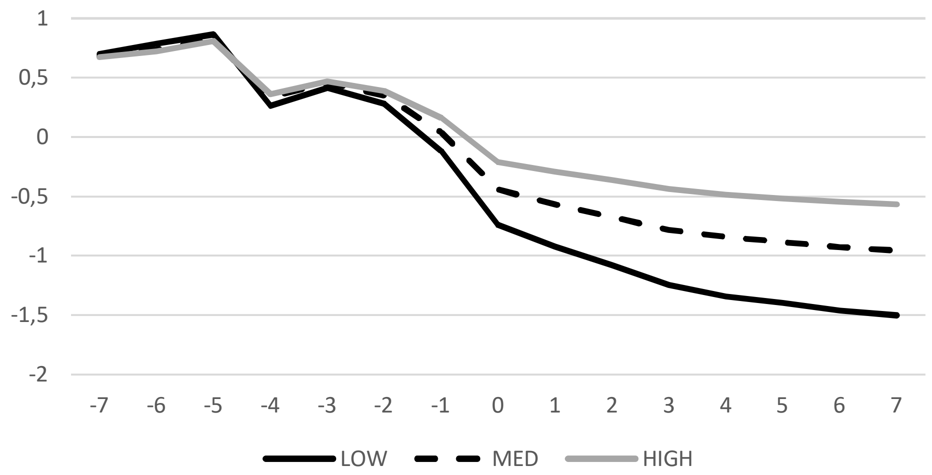


Fig. C.6 Policy 5